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UNIVERSITY OF ALBERTA

A NEW REFLECTION TRAVELTIME CALCULATION METHOD
AND ITS APPLICATION ON A SYNTHETIC VSP
TOMOGRAPHIC INVERSION STUDY

by

YANQUN WANG



A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE
OF MASTER OF SCIENCE
IN
GEOPHYSICS

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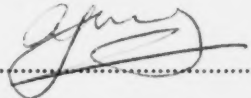
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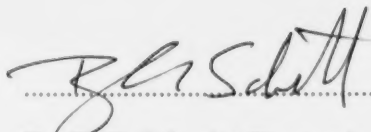
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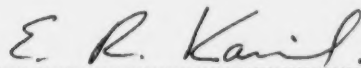
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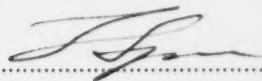
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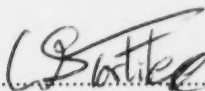
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ABSTRACT

In this paper an efficient and accurate reflection traveltime calculation method was developed and applied to a traveltime tomography study with special emphasis on applications in seismic monitoring of enhanced oil recovery in heavy oil reservoirs.

Modeling of seismic reflections can be difficult in regions of complex geology where standard ray tracing algorithms might be time consuming or even fail due to the existence of headwaves and shadow zones. An alternative approach is to map out arrival time to various portions of the geological structure and to in turn use this information to derive ray paths which might be useful, for example, in tomographic imaging. In the present study, we discuss a modification to Schneider et al.'s (1992) method to calculate reflection traveltimes and ray paths through complex geological structures. The reflection is treated as a constrained arrival traveltime computation in which the reflection traveltime to a given point in a medium from the reflector may be thought of as "the first arrival" of the wave which must emanate from this reflector. This technique calculates the reflected traveltimes by placing a series of Huygens' sources along the reflecting interface. These sources are activated when an arrival is detected at their position and their response is propagated back into the medium. Inclined horizons are modeled as a series of discrete steps. The maximum error observed is 1.6% of the total traveltime as derived from theoretical results calculated by straight forward application of Snell's law via ray tracing. The potential advantages of such a technique are the relatively rapid speed of calculation and the ease with which models might be updated for successive calculations.

A synthetic VSP tomographic study on a heterogeneous velocity structure similar to that encountered in the steam injection region in the shallow oil sands formations near Ft. McMurray was also carried out. The preliminary result shows that transmitted rays are severely refracted and even critically refracted by the high velocity carbonates which lie immediately below the oil sands. Therefore, it is impossible to image a region just above the carbonates using directly transmitted waves only; reflections must also be included. By applying the new technique, reflected rays are traced through a reference velocity structure. Our results show that the range of illumination angles on the required geological structure has been greatly improved and the vertical content of the structure is readily seen with the inclusion of the reflections although the horizontal boundaries are not well defined. The model misfit error E is 0.130 ms and the relative rms velocity misfit is 5.6% between the resulting image and the true model.

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Chapter 1 Introduction

This work mainly deals with the traveltimes of seismic waves, i.e., the time required for seismic waves to travel through the earth from the place they were produced to the point where they were detected. The purpose of this thesis is to develop an efficient and accurate reflection traveltime calculation method and apply the method to a traveltime tomography study with special emphasis on applications in seismic monitoring of enhanced oil recovery in heavy oil reservoirs.

In the geophysical monitoring of enhanced oil recovery processes, of environmental site reclamations, and certain mining applications, one of the promising approaches may be the application of seismic techniques to the remote sensing of geophysical anomalies. Information on the structure of the earth is contained in seismic waves which propagate through the earth. This wave propagation may fundamentally be described by the elastic wave equations (Aki et al., 1980) or the wave equations in porous media (Biot, 1956a, b; de la Cruz and Spanos, 1985, 1989). In principle, the study of the earth should proceed by a complete description of the earth in terms of its filtering effect on seismic energy.

In practice, however, most everyday problems in the analysis of a set of seismic data essentially reduce to an analysis of the traveltimes. Although some important information has been lost by abstracting seismic data into only a series of traveltimes, traveltime analyses still provide the basis for

routine interpretation of many seismic phases and a guide in the testing of more sophisticated methods.

Numerous efforts have been made to determine the traveltimes of propagating waves. The oldest methods are based on geometric optical theory (Born et al., 1965). At present, commonly used methods are asymptotic wave theory (Cerveny et al., 1977; Aki et al., 1980; Hron, 1984). More recently, various finite difference algorithms or techniques based on the fundamental Fermat's or Huygens' principles have been introduced (e.g., Vidale, 1988, 1990; Podvin et al., 1991; Moser et al., 1991; Van Trier et al., 1991; Schneider et al., 1992 and Coultrip, 1993). These methods provide in one way or another the traveltimes through a model geological structure.

An important tool in production seismic monitoring is traveltime tomography (Chiu et al., 1986; Macrides et al., 1988; Kanasewich et al., 1989; Stewart, 1991). The term tomography has been applied to many different kinds of seismic analyses, but strictly speaking seismic tomography means an analysis whereby the velocity structures of geological media are calculated from observed arrival traveltimes. Among the factors that may affect the accuracy of the resulting velocity structure, which may be taken as an "image" of the geological structure, is the accuracy of the paths of the propagating seismic energy, or rays, used in an inversion process, another is the angular coverage of rays on the required geological structure (Bregman et al., 1989b).

In a realistic geological structure, the velocity may vary both vertically and laterally. As a result, ray paths through the geological structure may be very complex and a large number of calculations must proceed in order to obtain the ray paths within an acceptable error bound (George et al., 1987;

Bregman et al., 1989a). This procedure constitutes a large part of the computational cost in ray tomography. Therefore, efficient methods to find ray paths are required in traveltime tomographic inversion.

As traveltime tomography is basically to invert for the physical properties, especially the velocity information from observed data, the angular coverage of rays on a geological structure may fundamentally affect the quality of resulting image (Bregman et al., 1989b). As a result, a dense ray coverage is required in traveltime tomography and it may be impossible to accurately reconstruct a velocity structure poorly covered by ray paths.

The primary motivation of this work is to attempt to image a heterogeneous velocity structure similar to that encountered in the steam injection region in the shallow oil sands formations near Ft. McMurray, Alberta. In this study, the background velocity structure and the position of a high velocity carbonate reflector are assumed to be already known to some extent in this well studied area (Carrigy, 1959). Two attempts are made here to improve the quality of a calculated velocity reconstruction. First, in order to increase ray coverage and use more information, observed reflection arrival traveltimes from the carbonate-oil sands interface are used in tomographic inversion. Second, a new efficient and accurate reflection traveltime calculation method is developed and reflected ray paths may then be easily found. This latter development makes reflection tomographic inversion applicable even for a complex geological structure.

In Chapter 2 of this thesis the theoretical bases are laid by reviewing the fundamental equations and principles that may be used to calculate traveltimes and to reconstruct velocity structures in tomographic inversion

procedures. A few commonly used numerical methods to calculate traveltimes are also reviewed in this chapter as these methods are germane to the present study.

In Chapter 3 the new reflection traveltime calculation algorithm is provided. The algorithm is based on Huygens' principle and developed by modifying Schneider et al.'s (1992) method which is used to calculate first arrival traveltimes. The performance of this algorithm, which includes the efficiency, the accuracy, and the error analysis, is then studied on a number of complex velocity models. A detailed review of this method is also presented in this chapter to aid in understanding this new algorithm.

In Chapter 4 the new algorithm developed in Chapter 3 is applied to a synthetic vertical seismic profiling (VSP) tomographic inversion study in combination with the new technique which has the advantage that ray paths may be easily determined. These two techniques (i.e., the techniques to calculate traveltimes and to find the subsequent ray paths) make curved ray tomographic inversion economic. The efficiency, the accuracy and the stability of some existing inverting algorithms are also studied on a hypothetical geological velocity model and the "algebraic reconstruction technique" is used for our final inversion process. Finally, a synthetic VSP tomographic inversion is carried out to image a heterogeneous velocity structure similar to that which might be expected in the steam injection region in the shallow oil sands formations near Ft. McMurray, Alberta.

Chapter 2 A Review of Commonly Used Numerical Methods in Traveltime Calculations

2.1 Introduction

As noted earlier, many problems in the analysis of a set of seismic data essentially reduce to analyzing the traveltimes. As a result, there exists a need for methods which allow calculation of the traveltimes that are approximations to a full wave theory. Asymptotic wave theory, or ray theory (Cerveny et al., 1977; Aki et al., 1980; Hron, 1984), as well as Fermat's and Huygens' principles (Born et al., 1965) are a few efficient approximations which describe the traveltimes of seismic waves and the propagation of seismic energy in geological media. They provide the basis for routine interpretation of many seismic phases and a guide in the testing of more sophisticated methods. Since ray theories are based on an approximation to the full elastic wave equations, their applicability are subject to a number of fundamental restrictions. The theoretical basis of calculating the traveltimes, the conditions under which application of ray theories is appropriate, and a few commonly used numerical techniques for obtaining the seismic traveltimes are reviewed in this chapter.

The oldest but still widely used ray method derives from geometrical optics theory (Born et al., 1965) which describes the physical characteristics of visible light in the language of geometry. Since the wavelength of the electromagnetic field associated with a propagating visible light is so small (in the order of 10^{-5} cm), it is therefore valid to completely neglect the finiteness

of the wavelength in many problems of optics. In this approximation, the propagation of wave energy is fully described as following along the paths of geometrical curves, or rays. The geometrical spreading of wave energy is described by a system of surfaces, called wavefronts (i.e., the spatial iso-lines of traveltimes) which are orthogonal to the rays in an isotropic medium.

Several advanced ray methods have been developed to improve the computational efficiency of traveltimes (see Ben-Menahem et al., (1985) for reviews). In these methods, the ray paths are always first obtained and the traveltimes are then calculated by integrating the slowness (i.e., the reciprocal of the wave's speed) along the ray paths (Cerveny et al., 1977; Aki et al., 1980).

More recently, workers have focused instead on the calculation of the first arriving traveltimes (e.g., Vidale, 1988, 1990; Podvin et al., 1991; Moser et al., 1991; Van Trier et al., 1991; Schneider et al., 1992 and Coultrip, 1993) by various finite difference algorithms or techniques based on the fundamental Fermat's or Huygens' principles (Born et al., 1965). In these methods traveltimes to a set of points over the entire structure are first calculated. The ray paths then follow the steepest descent directions of the wavefronts backwards from receivers to the source (Vidale, 1988, 1990). As these methods are germane to the present study they are reviewed in this chapter.

In the following sections of this chapter, the theoretical basis of the ray methods and its validity are first discussed. Two basic principles governing the ray and wave propagation, are then introduced. Finally, a review of the numerical methods of the principal ray and the traveltime calculation are provided.

2.2 The Validity of the Eikonal Equation

The geometric ray theory is a widely used, but approximate, technique in seismology. However, its applicability is subject to a number of fundamental restrictions. Three physical characteristics form the basis for these restrictions (Ben-Menahem et al., 1985): the wave frequency, the distance that the wave propagates, and the spatial gradients of the velocity and density of the geological structure of the medium. If any of the bounding conditions on these characteristics is exceeded, ray theory may not be valid and the full wave theory or a different approximation method must be used.

The geometrical ray method is based on the eikonal (which is derived from Greek for image) equation which governs the kinematics of wave characteristics. The eikonal equation describes how the traveltime of a propagating wavefront or a ray depends on the medium's velocity structure. Mathematically, the eikonal equation may be expressed as:

$$(\nabla t)^2 = \frac{1}{v^2} \quad (2.1a)$$

or is written explicitly in a Cartesian coordinate (x, y, z) as

$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial y}\right)^2 + \left(\frac{\partial t}{\partial z}\right)^2 = \frac{1}{v(x, y, z)^2} \quad (2.1b)$$

where $t(x, y, z)$ is the traveltime of a propagating wavefront to a point (x, y, z) and v is the wave speed for the medium. Note that v can be the compressional or the shear wave velocity. The detailed mathematical process of deriving the eikonal equation is contained in many texts and need not be repeated here (e.g., Born et al., p112, 1965; Aki et al., p90, 1980; Cornbleet, 1983; Hron, 1984; Kravtsov et al., p6, 1990). Briefly, however, equation (2.1) may be

derived from the full wave equation in a weak inhomogeneous medium under a high frequency approximation by retaining the first term in asymptotic series which is an expansion of the different order of the spatial derivatives of the wave amplitude and the phase (Cornbleet, 1983; Hron, 1984; Kravtsov et al., 1990). The asymptotic series may be written as

$$\begin{aligned}\psi(\mathbf{x}, t) &= \exp[i\omega t(\mathbf{x})] \sum_{n=0}^{\infty} \frac{(2\pi i)^n}{n!} \left[\frac{\partial^n A(\mathbf{x}, t)}{\partial t^n} \right]_{t=t_0} \frac{1}{(i\omega)^n} \\ &= \sum_{n=0}^{\infty} A_n(\mathbf{x}) f_n(t)\end{aligned}\quad (2.2)$$

where $\psi(\mathbf{x}, t)$, $A(\mathbf{x}, t)$, ω and $t(\mathbf{x})$ are the displacement, the amplitude, the frequency of the wave, and the travelttime to a given point $\mathbf{x}$, respectively. The terms $A_n(\mathbf{x})$ and $f_n(t)$ in equation (2.2) may be expressed as

$$A_n(\mathbf{x}) = \frac{(2\pi i)^n}{n!} \left[\frac{\partial^n A(\mathbf{x}, t)}{\partial t^n} \right]_{t=t_0} \quad (2.3a)$$

$$f_n(t) = \exp[i\omega t(\mathbf{x})] \frac{1}{(i\omega)^n} \quad (2.3b)$$

Three conditions immediately arise from the derivation of equation (2.1) and the subsequent characteristics of the eikonal equation (Ben-Menahem et al., 1985).

(1). The P and S wave motions must be fully separate. The velocity in eikonal equation (2.1) represents either the P or the S wave velocity, and fully separate P and S wave fields are required. This can be a problem due to the fact that the particle motions for P and SV waves are coupled. For example, a pure compressional wave incident upon a geological interface separating materials of differing velocities may reflect and refract from this interface as

both P and SV phases. However, it has been shown (Ben-Menahem et al., 1985) that for weak inhomogeneous media where the velocities and density change slowly over distances of the order of wavelength, the coupling term between the P and SV wave motions in the wave equations may be ignored and the approximation of fully decoupled P and SV wave fields is valid. That is, a pure phase will not produce the other whilst propagating. This condition may be expressed mathematically as (Ben-Menahem et al., 1985)

$$\omega \gg \omega_0 \quad (2.4a)$$

$$\omega_0 = \frac{1}{2} \max(|\nabla\alpha|, |\nabla\beta|, \alpha \frac{|\nabla\rho|}{\rho}) \quad (2.4b)$$

where ω , α , and β are the wave frequency, and the P and S wave velocities, respectively. ρ is the density of the medium and ω_0 is called the threshold frequency and is representative of the characteristic rate of change of the physical properties of the medium given by equation (2.4b).

(2). The condition of a high frequency approximation ($\omega \rightarrow \infty$) is required if waves can be described as geometrical rays which are governed by the eikonal equation (Born et al., Chapter 3, 1965). Under this approximation, all other wave properties may be disregarded and the propagation of wave energy is fully described as the paths of rays. In practice, however, this high frequency approximation may be assumed valid when (Ben-Menahem et al., 1985)

$$\omega \gg \omega_c \quad (2.5a)$$

$$\omega_c = v(\nabla^2 A_0/A_0)^{1/2} \quad (2.5b)$$

where v is the medium velocity and A_0 is the amplitude term of the first term in asymptotic series (2.2) (Cornbleet, 1983; Hron, 1984; Kravtsov et al., p6,

1990). The inequality (2.5a) and equation (2.5b) indicate that the amplitude A_0 must vary slowly over the scale of a wavelength. It may also be shown that under this approximation the dispersion effect for a propagating wave is a second order effect and may be ignored (Ben-Menahem et al., 1985).

(3) The final condition is referred to as the far-field or radiation zone restriction. As the eikonal equation (2.1) is an approximation to the wave theory simplified by retaining only the first term in asymptotic series (2.2) (Cornbleet, 1983; Hron, 1984; Kravtsov et al., p6, 1990), the cumulative error produced by ignoring the higher order terms in equation (2.2) must be sufficiently small that the geometrical ray theory remains a correct quantitative approximation. This gives the upper bound on the source-receiver distance which is called the radiation zone (the region in which the geometrical ray theory is valid) condition or the far-field restriction (Ben-Menahem et al. 1985) which is written as

$$t \ll \omega \omega_c^{-2} \quad \text{or} \quad \omega t \ll \omega^2 \omega_c^{-2} \quad (2.6)$$

The upper bound on source-receiver distance is indicated by the traveltime t in inequality (2.6). This time domain distance limitation increases with the increase of the wave frequency ω and the decrease of the characteristic frequency ω_c in the high frequency approximation (2.5). The largest scale of the radiation zone is limited by the wave frequency of concern. When an infinite frequency exists ($\omega \rightarrow \infty$), there is no spatial limitation to the applicability of ray theory.

It should be noted that there are alternative forms to the eikonal equation which are more often used in practice. For example, taking $\hat{l}_0$ as an

instantaneous unit vector tangent to a ray at time $t(x, y, z)$ (Figure 2.1), the eikonal equation (2.1) may be rewritten as

$$\nabla t \cdot \hat{l}_0 = \frac{1}{v} \quad \text{or} \quad \frac{dt}{dl} = \frac{1}{v} \quad (2.7)$$

where dl is an infinitesimal ray segment on a ray path. This representation is advantageous in that integrating equation (2.7) along the ray path yields the traveltimes to a given point

$$t(\mathbf{x}) = t_0(\mathbf{x}_0) + \int_{l_0}^l \frac{dl}{v} \quad (2.8)$$

where $t(\mathbf{x})$ and $t_0(\mathbf{x}_0)$ represent the arrival time for a wavefront to points $P(\mathbf{x})$ and $P_0(\mathbf{x}_0)$ (Figure 2.1), with $l(\mathbf{x})$ and $l_0(\mathbf{x}_0)$ being the lengths of the ray path from a certain point (e.g., the source point indicated by the star in Figure 2.1) to these two points, respectively.

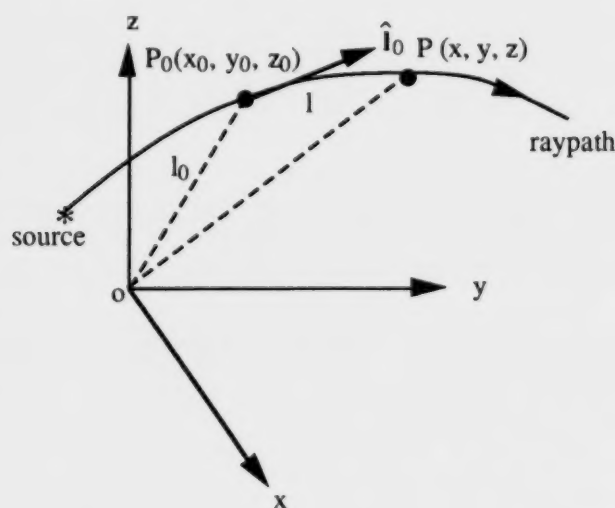


FIG. 2.1. Ray cartoon. l_0 and l are the lengths of the ray paths from the source to point P_0 and P , respectively. $\hat{l}_0$ is an instantaneous unit vector tangent to the ray.

The eikonal equation (2.1) forms the basis of both geometrical ray theory and recently developed finite difference traveltime calculation methods. Equations (2.1) and (2.8) show that the wave traveltimes are completely governed by eikonal equation under the high frequency approximation.

2.3 Fermat's and Huygens' Principles

Parallel to the eikonal equation, there are two basic principles that deal with wave and geometrical ray properties. They are Huygens' and Fermat's principles, respectively.

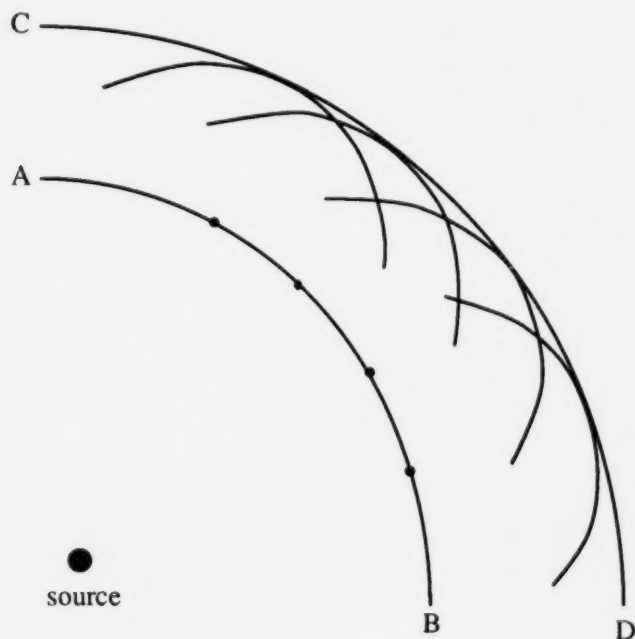


FIG. 2.2. Huygens' principle. Wavefront CD is determined when AB is given. Every point on wavefront AB (e.g., the small solid circles) acts as a secondary source.

Huygens (Born et al., 1965) proposed a wave theory which predicts the future position of a wavefront when its position at earlier time is known. Huygens' principle is stated as follows: every point on a wavefront is itself considered as a new source of tiny wavelets that spread out in the forward

direction at the speed of the wave itself. The new wavefront is the envelope of all these wavelets (Figure 2.2). Huygens' principle is a particularly useful concept to apply when the wavefronts are interrupted by discontinuities in material properties, especially the elastic impedance which produces both diffractions and reflections. In our present studies, Huygens' principle forms the basis of the newly developed reflection method which will be discussed extensively in Chapter 3.

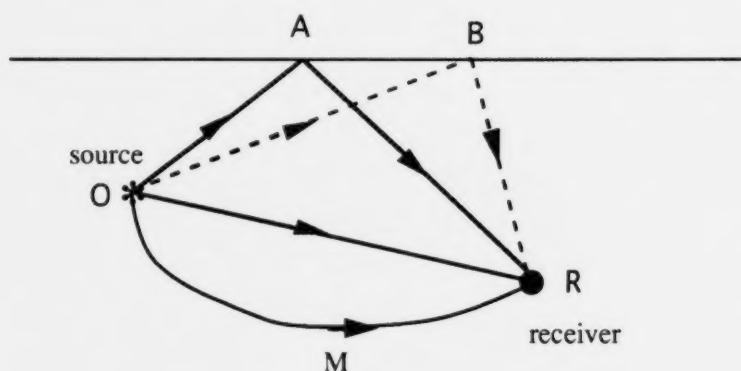


FIG. 2.3. Fermat's principle. Ray paths from O to R are straight line O-R and line O-A-R which make the traveltime between source and receiver stationary.

The second important principle implicit in all traveltime calculations is Fermat's principle. Fermat's principle is widely used in ray theory and is stated as follows: the ray paths between two points are those along which the traveltimes from one point to another are a minimum or a maximum with respect to all other nearby possible paths (Figure 2.3). Fermat's principle controls ray behaviors of all kinds; indeed, it is the basis of Snell's law which describes how ray paths change their directions through media of varying velocities. Fermat's principle is also a basis of Schneider et al.'s (1992) method

which calculates the traveltime of the first seismic energy at any given point within the medium.

2.4 Solving the Eikonal Equation in a Ray Co-ordinate

— the Conventional Ray Tracing Methods

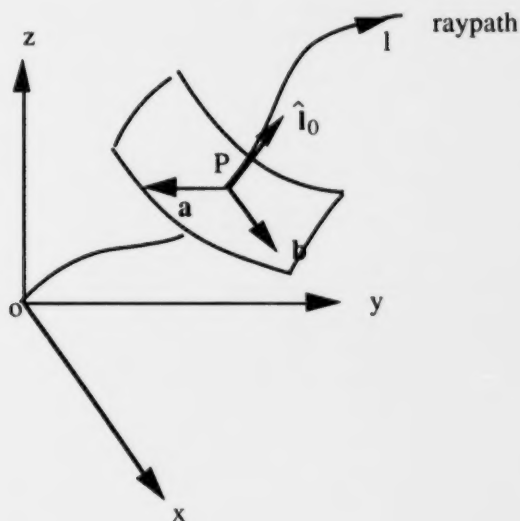


FIG. 2.4. Ray co-ordinate system cartoon. $\hat{l}_0$ is an instantaneous unit vector tangent to the ray l at point P . It is also the normal vector of the wavefront at this time. $\mathbf{a}$ and $\mathbf{b}$ are two arbitrary unit vector orthogonal to each other in the wavefront.

Note that a ray describes the path of the propagation of seismic energy through a structure. In an isotropic medium, a system of surfaces orthogonal to the rays are called wavefronts (Aki et al., 1980). A special co-ordinate system can be set up by taking an instantaneous unit vector $\hat{l}_0$ tangent to a ray at point P (Figure 2.4) in combination of another two arbitrary unit vectors $\mathbf{a}$ and $\mathbf{b}$ orthogonal to each other in the wavefront at this point (Hron, 1984). The ray coordinate system is based on the three orthogonal vectors and the process of solving the eikonal equation in this system is known as ray tracing. However, in most practical cases where the starting and the ending points are

already given and the rays must propagate through a complex geological structure, the eikonal equation (2.1) is too complicated to be solved directly either analytically or numerically. Instead, two approaches to the problem are widely used in practical applications and they are referred to as the shooting and the bending methods (Aki et al., 1980).

The shooting method is an initial value problem. Given the initial position and direction of a ray, the end position of the ray is found by integrating the ray equations. The ray converges to the desired end point by perturbing the initial direction (Figure 2.5 (a)). Therefore the shooting generates a fan of rays and the one which reaches the receiver will be chosen. However, diffracted ray paths and the ray paths in shadow zones will not be found as for these situations geometrical ray theory does not hold.

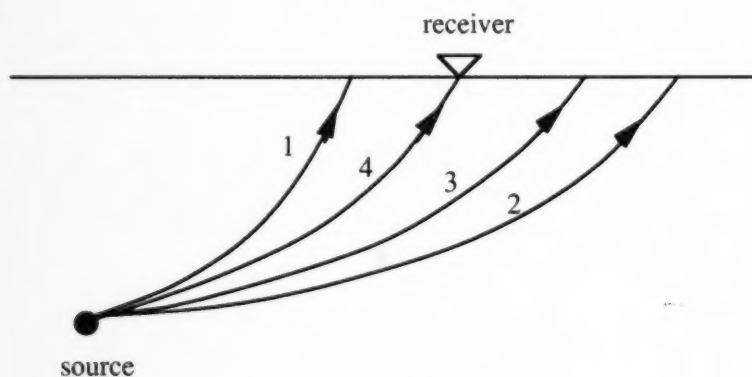


FIG. 2.5 (a). The shooting method. Starting from the source (the solid circle), a trial ray (1) is propagated towards the receiver (the triangle) using the ray equations. If the trial ray misses the receiver, the initial ray direction is perturbed and new trial rays are traced (2 and 3). This procedure iterates until the target is reached (4).

The bending method takes the direct boundary value problem approach. In this method, an initial path is chosen according to the best previous knowledge, iterative perturbations are then computed in order to derive the

ray path associated with the minimum time (Figure 2.5 (b)). However, as there are infinite numbers of ray paths connecting two points of interest in a medium, in integrating to determine a total traveltimes it is quite possible that a ray associated with a local minimum is chosen instead of the true ray path with the global minimum. This situation arises for a complex geological structure where critically refracted rays, or head waves, are produced at geological interfaces. Therefore the initial choice of a ray path is important to this method as it affects the speed of convergence of the calculations and the accuracy of the first arrival traveltimes obtained.

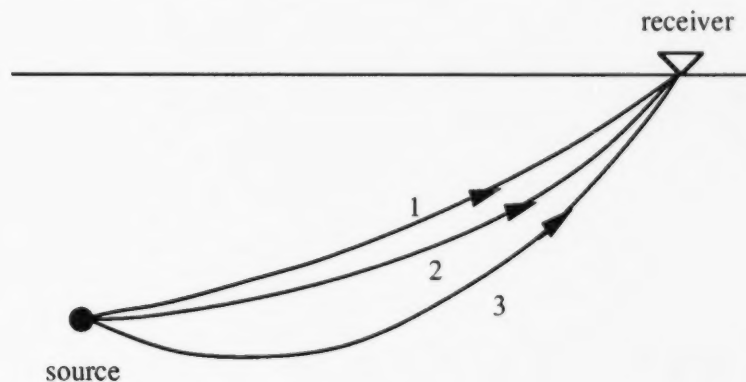


FIG. 2.5 (b). The bending method. An initial path estimate (1) is perturbed (2) until it satisfies the ray equation (3). Successive perturbations are calculated from a perturbation analysis of the ray equations.

It should be noted that both of these techniques essentially require repetitively calculating over a number of possible ray paths until one is found which falls within acceptable error bounds. This can result in long computational time (and cost) and makes the techniques impractical if a large number of rays are required or if the calculations are carried out through a geologically complex medium. As a result, there remain numerous practical requirements for a forward seismic modeling method which can obtain ray

paths and traveltimes associated with the first arrival energy both quickly and accurately relative to these often used conventional ray tracing methods. Numerous efforts have been made so far and the resulting methods will be briefly reviewed in the following sections.

2.5 Solving the Eikonal Equation Using Finite Difference Method

—Vidale's First Arrival Traveltime Calculation Method

As noted in the last section, there are two main difficulties with the above conventional ray tracing methods; they are computationally intensive and the obtained first arrival traveltimes are ambiguous in a complex geological structure. Although the latter may be overcome by a few advanced techniques (Doornbos, 1988), the difficulty of long computational time still remains. Therefore, there are practical needs for a faster and more accurate scheme for ray tracing and traveltime calculation, especially in complex geological media which can vary both laterally and vertically. For example, in curved ray traveltime tomographic inversions, a large number of individual rays must be calculated through complex geology (George et al., 1987; Bregman et al., 1989a).

To overcome these difficulties, Vidale (1988, 1990) proposed a first arrival traveltime calculation method which uses a finite difference solution to the eikonal equation. In his method, a velocity structure is sampled at equally spaced discrete points. The first arrival traveltimes (i.e., the wavefronts) to given points are obtained over the entire space. This differs from the above ray tracing methods which calculate times only along the rays. Ray directions are then obtained by following the steepest descent direction of the wavefronts backwards from receivers to the source. This method has obvious

advantages over the conventional ray tracing methods in complex media or when a large number of rays are required in that the ray tracing requires no iteration. Moreover, arbitrary recording geometries are easily accommodated since the arrival times are obtained everywhere throughout the geological structure. Therefore, this traveltimes calculation scheme and the associated ray tracing method are particularly useful in traveltimes tomographic inversion where the source-receiver combinations can be complex.

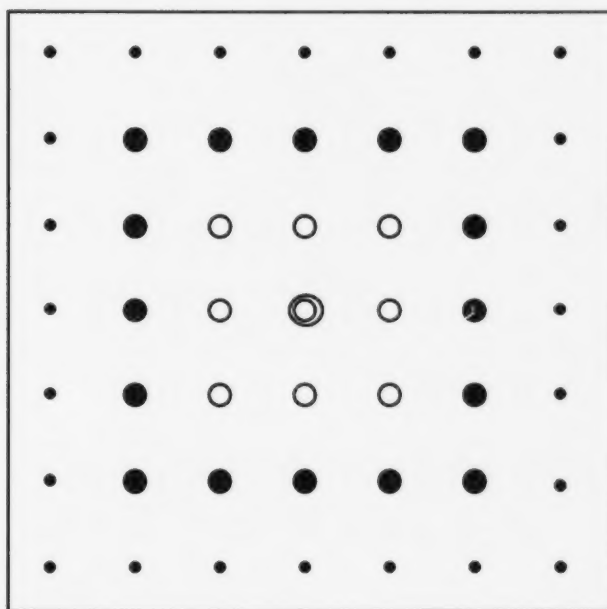


FIG. 2.6. Vidale's expanding square traveltimes calculation algorithm. Traveltimes to the points on a square ring are calculated from its immediate inner square.
(After Vidale, 1988)

Vidale's (1988) method may be described schematically as an expanding square algorithm as illustrated in Figure 2.6. Assume that the traveltimes to the points represented by the large solid circles on a square ring are to be calculated (Figure 2.6) and that the traveltimes to the points represented by the open circles within the square ring are already known. The traveltimes to

the points represented by the small solid circles are not calculated until the next immediate inner ring of solid circles has been calculated. The double open circle in the middle of the structure represents the source position. For simplicity, only a two-dimension space is shown here. Note that the traveltimes to the ring of solid circles are calculated from its immediate inner ring on which the traveltimes are already known (Vidale, 1988). During the calculation process, the local minimum traveltimes to the points on this solid ring are first calculated. The remainder are then calculated from both their adjacent local minima and the inner ring. After traveltimes to this ring are obtained, the calculation proceeds to the next immediate outer ring. Applying this algorithm (expanding squares), the entire grids are filled with traveltimes.

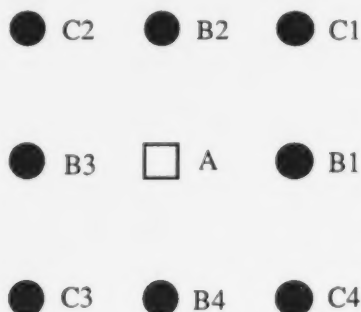


FIG. 2.7. Finite difference mesh centered about source point A, where B_i and C_i are to be timed (after Vidale, 1988).

During the above calculation process, finite difference approximations of a plane wave are applied to the differential terms in the eikonal equation (2.1) which may be written explicitly in a 2-D Cartesian coordinate system as:

$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial z}\right)^2 = s(x,z)^2 \quad (2.9)$$

The two differential terms in equation (2.9) may then be approximated explicitly with finite difference as (Figure 2.7)

$$\frac{\partial t}{\partial x} = \frac{1}{2h} (t_A + t_{B_{i+1}} - t_{B_i} - t_{C_i}) \quad (2.10a)$$

$$\frac{\partial t}{\partial z} = \frac{1}{2h} (t_A + t_{B_i} - t_{B_{i+1}} - t_{C_i}) \quad (2.10b)$$

where t_A , t_{B_i} , and t_{C_i} are the traveltimes to points A , B_i , and C_i ($i=1, 2, 3, 4$), respectively, h is the mesh spacing and $t_{B_5} = t_{B_1}$.

Vidale's finite difference method is highly accurate in calculating first arrival traveltimes for media with relative low contrast in velocities throughout the structure. The relative errors are estimated to be at most 0.1 percent (Vidale, 1988). The arrival traveltimes are returned correctly while conventional ray tracing may miss some of the first arrivals. Traveltimes are also obtained everywhere throughout the model rather than only along the ray paths. The iso-time lines of these traveltimes are the wavefronts and they may penetrate into shadow zones and also account for head waves along interfaces. Vidale's method is computationally economic as well. It can be 20 to 60 times faster than a conventional ray tracing method, depending on the dimensions of a model (Vidale, 1988).

However, there are two limitations to Vidale's (1988, 1990) method. First, a plane wave approximation is used in this method; this condition is not necessarily valid for the wave field near the source or at a sharp interface. Although Vidale (1988) proposed a circular wavefront calculation scheme in order to model high curvature cases, there are no a-priori criteria indicating under which conditions the curved wavefront scheme should be used. Second, an implicit assumption is that the traveltime to a point on a square

ring can be obtained from the locally adjacent points on the prior inner square at each step (Figure 2.6). As a result, causality is often violated with this expanding square strategy in media with high velocity contrast (e.g., ratio of high velocity/low velocity $> \sqrt{2}$ for an interface (Qin et al., 1992)). An example describing a case where the expanding square algorithm is invalid is shown in Figure 2.8. A two layer model is considered in this case and the source is located at point S in the low velocity layer. The actual first arrival to point C is the head wave and comes along the path S-A-B-C. However, in this expanding square algorithm the traveltime to point C has already been calculated (i.e., along the path S-C) before the square ring reaches the interface. Therefore, the effect of the high velocity layer is not included and this results in the inaccurate first arrival traveltime to point C.

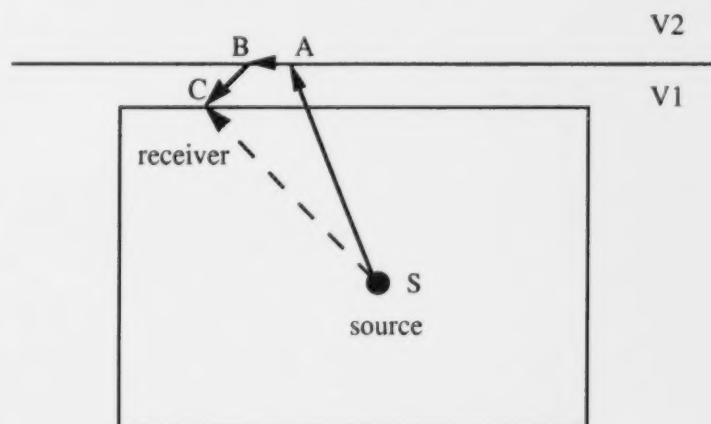


FIG. 2.8. A two layer example to show the invalidation of the expanding square method, where S is the source point and C is the point to be timed. V2 and V1 are the velocities of the two layers and $V2 > V1$ (after Qin et al., 1992).

2.6 Other First Arrival Traveltime Calculation Methods

Inspired by Vidale's early work (Vidale, 1988, 1990), others have developed additional techniques to calculate the first arrival traveltimes more easily, faster and more accurately.

Qin et al. (1992) modified Vidale's method on the basis of Huygens' principle. Instead of using the expanding square algorithm, they calculated traveltimes along expanding wavefronts; this has the advantage that causality is always obeyed. Instead of identifying the points associated with local minimum traveltimes along an edge of a solution square, in Qin et al.'s (1992) method the global minimum traveltime point along the solution boundary is identified and the traveltime to this point is first calculated at each step. Then the finite difference solution region is expanded starting from this global minimum point. However, because a massive search for the global minimum is required at each step, it is computationally expensive with the number of calculations increasing roughly as the cube of the number of grid points (Qin et al., 1992).

To remedy the process of the search for the local minimum points in Vidale's (1988) method and to increase the computational efficiency, Van Trier and Symes (1991) solved an alternative form of the eikonal equation from the fluid dynamic point of view. A different finite difference method called the upwind finite difference approximation technique is applied in their algorithm. Their finite difference process is fully vectorized so that more powerful computers can be used. However, maintaining causality is a problem as in Vidale's method.

Unlike the above methods, which may be considered as variations of an expanding ring technique (Vidale, 1988, 1990; Qin et al., 1992; Van Trier et al., 1991), Podvin and Lecomte (1991) focused on the computation of the arrival traveltimes to a given point from its adjacent point. Their algorithm was implemented in a massively parallel fashion for speed requirements. Based on Huygens' principle and the planar wave finite difference approximation to the eikonal equation, the traveltime to a given point of a medium is estimated from the known arrival times at each of its neighboring points. For example, in the two dimensional case (Figure 2.9), the traveltime to the center point (the point with open circle) is defined to be the least of the calculated values from the known traveltimes at the eight neighboring points which are represented by solid circles. The traveltimes at these solid circles are assumed either from the initial guess or from the previous calculation result. During an elementary iteration, all grid points simultaneously compute their arrival time according to the state of their nearest neighbors. This process is repeated until no further update is possible and traveltimes to all the grid points in the space are minima (Podvin et al., 1991). In this algorithm, the number of iterations required is of the same order as the maximum distance (in mesh units) from the source to the model boundaries. The relative error of the method is estimated to be at most 0.1% and this error depends on the mesh size and velocity contrasts within the structure. Note that their algorithm is developed for the implementation on a parallel computer which is not always available and this limits its application in some sense.

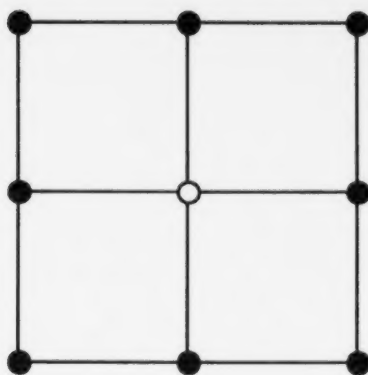


FIG. 2.9. The calculation of the first arrival traveltimes to the current point (the point with open circle). Surrounding points (points with solid dots) are considered as a set of Huygens' sources (After Podvin and Lecomte, 1991).

Whilst most workers apply their efforts in solving the eikonal equation more accurately and efficiently, Moser (1991) provided a different way to calculate the first arrival traveltimes which is based on a subset of mathematics called graph or network theory. In this scheme, discrete points in a model are connected by arcs which represent a certain kind of weighted physical property, as, for example, the distance weighted by the slowness between two points in a ray tracing problem. Each point is connected to a limited number of its neighboring points by these arcs. It is therefore possible to travel from one point to another via the arc connections and different arc connections between the two points give different traveltimes. The shortest traveltime connection between the two points is obtained with the help of graph theory. The detailed calculation algorithm is given by Moser (1991). One advantage of Moser's method is that all shortest paths from one point are constructed simultaneously. This is particularly advantageous when applied to those cases associated with common-shotpoint gathers: a geometry used in exploration seismics. The total number of operations is of the order

of $n \log n$, where n is the number of grid points in the structure. The accuracy of this method is quadratically dependent upon the number of points per coordinate direction and the number of connections per point. Moser's method actually is an alternative application of Fermat's principle to network theory.

Schneider et al. (1992) also provided an algorithm that essentially relies on the basic principle that the first arriving energy received at a given point must come through the adjacent surrounding medium. Consequently, all possible arrival times of energy to a given point from all directions are first calculated. The first arrival traveltimes to the point is then obtained by seeking the minimum of these values. Similar to Podvin et al.'s (1991) method, Schneider et al.'s (1992) method focuses on the computation of the local minimum traveltimes, i.e., traveltimes to a given point is calculated accordingly to its neighbors. In Schneider et al.'s method Fermat's and Huygens' principles are the theoretical bases for traveltimes calculations whereas a finite difference solution to the eikonal equation and Huygens' principle are used in Podvin et al.'s (1991) method. Schneider et al.'s method is easily coded and the causality problem occurring in Vidale's (1988, 1990) and Van Trier et al.'s (1991) methods are remedied since Huygens sources are introduced. This technique is later modified in this paper for calculating reflection traveltimes. As Schneider et al.'s method is one of the basis of the new reflection method, it will be presented in detail in Chapter 3.

There are numerous other efforts in traveltimes computations. For example, to improve the computational accuracy, Asakawa et al. (1993) used dense discrete points which are distributed both on the corners and along the edges of grids whereas Coultrip (1993) and Li et al. (1993) used triangle grids to

model interfaces. However, all these efforts are more or less based on Vidale's (1988) early work in which first arrival traveltimes are obtained through wavefront propagation.

2.7 The Restrictions of the Eikonal Equation

There are some basic difficulties with ray tracing methods. For many complex geological structures, rays traced from a source can never reach certain regions of a medium (e.g., shadow zones) where the signals that are received are really diffractions from some localized secondary sources (Huygens principle) which are not described by Snell's law. Simple finite difference solutions to the eikonal equation do not fully solve this problem (Vidale, 1988, 1990; Van Trier et al., 1991) since this theory is based on the geometrical optics approximation to the wave theory. In the ray tracing methods, an expensive and explicit diagnosis of diffractions must be implemented in these shadow zones in order to account for them (Doornbos, 1988). In finite difference approximation schemes, precautions must be taken to avoid the mathematically conflicting situation of a square root of negative values which can occur when the velocity contrast is too large for an interface (Qin et al., 1992). The reason for this is very simple; the change of a mathematical method does not change the nature of physical principles, but may only provide a more efficient tool. Signals picked up in shadow zones are diffracted by the secondary sources (diffractors), whereas a solution to the eikonal solution implicitly consists in a single propagating wavefront in a medium. Because of these diffractions, Huygens principle must be used in combination with the eikonal equation (Podvin et al., 1991; Qin et al., 1992) to account for all the important seismic arrivals.

Chapter 3 An Efficient and Accurate Reflection Traveltime Calculation Method

3.1 Introduction

In this chapter a new method for the calculation of reflected wave traveltimes is outlined. Since this newly developed reflection traveltime calculation method is based on Schneider et al.'s (1992) first arrival traveltime calculation algorithm, a detailed review of the algorithm is presented first in this chapter. A description of the reflection traveltime calculation method and a study of its performance for a number of complex velocity models follows.

Recently, Schneider et al. (1992) proposed a two-dimensional forward modeling traveltime calculation technique which is based simply on Fermat's principle. Unlike other first arrival traveltime calculation methods (e.g., Vidale, 1988, 1990; Podvin et al., 1991; Van Trier et al., 1991 and Coultrip, 1993), the present method uses a circular interpolation of wavefronts to determine the first arrival traveltimes over a discretized velocity model. This method has three principal advantages over other techniques in the calculation of first arrival traveltimes and these advantages broaden the circumstances under which it may be applied. First, more accurate and realistic circular wavefronts are used in the interpolation of the traveltimes, this contrasts with the plane wave approximation employed by most other workers (e.g., Vidale, 1988, 1990; Qin et al., 1992; Moser et al., 1991 and Van Trier et al., 1991). Compared with the other planar wave interpolations, the

higher order interpolation scheme in Schneider et al.'s (1992) method increases the calculation accuracy with little extra computing time.

Second, Schneider's method focuses on the computation of the local minimum traveltimes. In this method the minimum arrival traveltime is sought for each of the discrete points according to the arrival traveltimes to their neighbors. This contrasts with other methods such as finite difference where only the propagation of the leading wavefront is determined. Schneider et al.'s (1992) method is essentially based on the fundamental principle that the first arriving energy received at a given point in a structure must arrive via the adjacent surrounding medium. Consequently, the first arrival traveltime to the given point is obtained by seeking the minimum time by testing all the possibilities for arrivals of energy to the point from all directions. This change of calculation procedure avoids potential difficulties of causality which can arise in other techniques; for example, Vidale (1988) noted that "the time for the part of the ray path leading to a point must be known before the time of the point can be found". This problem appears in Vidale's (1988, 1990) and Van Trier et al.'s (1991) methods as noted earlier in Chapter 2 (see Figure 2.8).

A third advantage is that Schneider et al. (1992) use a simple discrete velocity model. That is, instead of sampling a continuous velocity function at discrete points (e.g., Vidale, 1988, 1990; Qin et al., 1992), the velocity model constructed by Schneider et al. (1992) consists of a set of rectangular blocks of constant velocity. This velocity model is useful and conforms automatically to the configurations used in many tomographic modeling schemes (e.g., McMechan, 1983; Macrides et al., 1988; Kanasewich et al., 1989).

Seismic reflection profiling forms the basis for a large global industry which seeks to aid the discovery of petroleum and mineral products. However, modeling of reflections can be difficult especially in regions of complex geology where standard ray tracing algorithms fail as noted earlier in Chapter 2. More lately seismic reflection has become one important issue both for purposes of seismic tomography (in the case of reflection tomography (Chiu et al., 1986)) and in seismic migration (Mora, 1989). For these reasons it is natural to extend Vidale's (1988) technique to the computation of reflection traveltimes. Treating the reflection as a constrained first arrival in the traveltimes computation is a promising idea as the reflection traveltime to a given point of a medium can also be thought of as "the first arrival" of the wave which must pass through the reflector. For example, Matsuoka et al. (1992) developed a reciprocity method in which the reflection traveltime is treated as the sum of two separate first arrival traveltime calculations. The first set are traveltimes generated from the real source to the reflecting interface and the second are from the surface receiver to the reflecting interface. Head waves are excluded in the traveltime calculation as the reflected wave propagates only in the medium above the reflecting interface. After the first arrival traveltimes are calculated using Vidale's (1988) method for each of the two "sources", the two sets of traveltimes are simply added together to give the complete traveltime. As stated by Fermat's principle, the reflection points for a given source-receiver pair are found by searching through all the possible points along the interface and picking those with minimum traveltimes. Depending on a geological model structure, multiple reflection points are possible for a given source-receiver pair (e.g., a syncline shaped reflecting interface). Although Matsuoka et al.'s (1992) method is a correct alternative technique for reflection traveltime calculations, the

method is computational time consuming as only a single traveltime for one source-receiver pair is calculated at each step.

In the following sections, Schneider et al.'s (1992) method is first reviewed and then here modified to allow calculation of the reflection traveltimes. Numerical examples of this technique for a number of complex geologies and corresponding error analyses are also provided.

3.2 Schneider's First Arrival Traveltime Calculation Method

3.2.1 The Computation of Local Minimum Arrivals

As stated earlier in this chapter, Schneider et al.'s (1992) method focuses on the computation of the local minimum traveltime, i.e., the traveltime to a given point is calculated according to the arrivals to its neighbors; the first arrival traveltime to the point is then obtained by seeking these local minimum traveltimes arriving from all the possible directions. The theoretical basis for a minimum arrival time calculation is provided in this section, the minimum time search technique is presented in the following section.

For simplicity, assume that rectangle ADQM as shown in Figure 3.1 is the region of interest over which traveltimes are to be calculated. This rectangle is divided into six smaller rectangles denoted 1 to 6 (Figure 3.1). Within each of these rectangles, or more aptly termed cells, the velocity is constant. Each corner of a cell is called a node and it is the traveltime to these nodes which are to be determined. The star in Figure 3.1 is the source position which is taken to be the same as the origin of x - z Cartesian coordinate grid. As the velocity is constant within each cell, the path of energy

propagation (i.e. the ray) from one side of the cell to the other must follow a straight line. No refraction takes place within any cell.

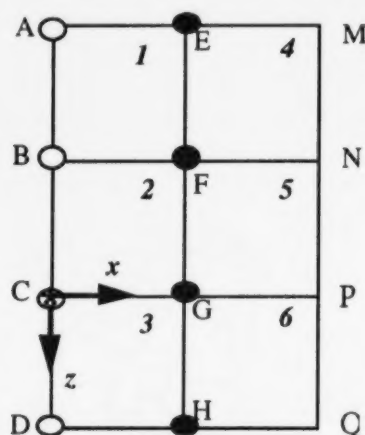


FIG. 3.1. A simple geometry for traveltimes calculations. The traveltimes to grid points are to be calculated. The velocity is constant within each cell. The source is located at the grid point represented by the star.

Figure 3.2 shows one cell ABCD of horizontal and vertical sides of length Δx and Δz , respectively. At our starting point for calculations across this cell, the traveltimes t_1 and t_2 to points A and B, respectively are already known. According to Fermat's principle, the first arriving seismic energy to node $D(x+\Delta x, z_2)$ at time t through the edge connecting nodes A and B is obtained by minimizing t with respect to z_0

$$t = t_0 + S \cdot [(z_2 - z_0)^2 + \Delta x^2]^{\frac{1}{2}} \quad (3.1)$$

Here S is the slowness (i.e., the reciprocal of the wave speed) of the cell; z_0 is the z -component coordinate of the intersection point P on the cell edge connecting A and B where the ray enters the cell at arrival traveltime t_0 as shown in Figure 3.2.

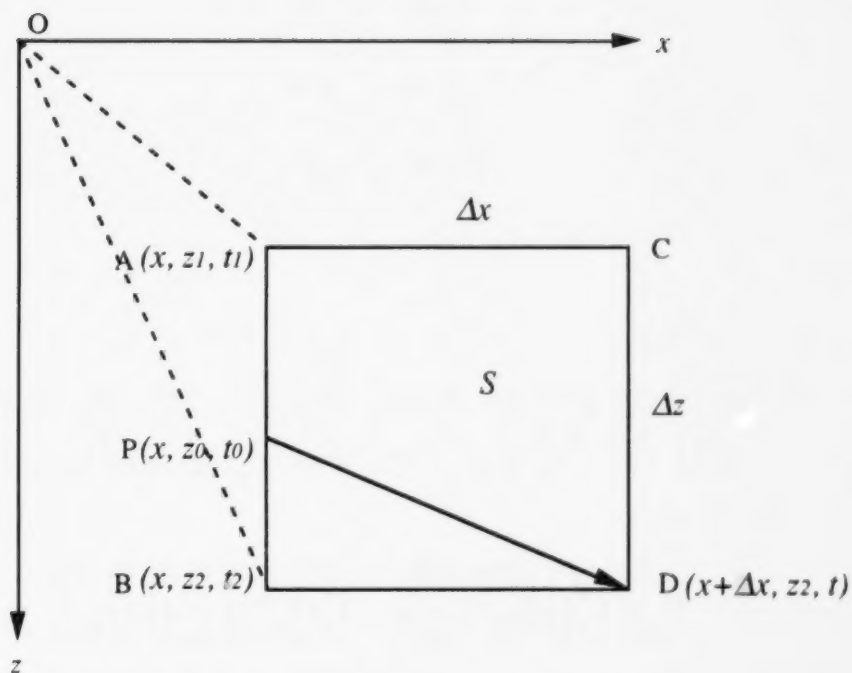


FIG. 3.2. Local minimum traveltime calculation scheme. The first arrival to grid point D through edge AB is calculated by locating point P through which the traveltime t to point D is the minimum according to Fermat's principle.

A high order circular wavefront interpolation is used to determine t_0 in equation (3.1). It is known that wavefronts generated by a point source are circular if a medium has uniform velocity. For a source located at the origin O (0, 0) of the co-ordinate system in Figure 3.2, the time-distance relationships for points A (x, z_1) and B (x, z_2) are, respectively

$$t_1^2 = S_a^2 \cdot (x^2 + z_1^2) \quad (3.2a)$$

$$t_2^2 = S_a^2 \cdot (x^2 + z_2^2) \quad (3.2b)$$

where S_a is the slowness of the medium. Eliminating x between equations (3.2a) and (3.2b), yields:

$$S_a^2 = (t_2^2 - t_1^2) / (z_2^2 - z_1^2) \quad (3.3)$$

Using a simple linear interpolation formula in squared quantities, the traveltimes to point P (x, z_0) (Figure 3.2) satisfies

$$t_0^2 = S_a^2 \cdot (z_0^2 - z_1^2) + t_1^2 \quad (3.4)$$

When a medium slowness is variable, equation (3.4) becomes a high order interpolation (compared to the planar wave interpolation) for the traveltimes to point P. In this case S_a in equation (3.3) is the apparent slowness which would be determined if one could observe only the traveltimes at points A and B. In order to obtain the first arrival traveltimes to point D, z_0 , which is the z component of the intersection point P, must be calculated using Fermat's principle. That is, the intersection point P must be the point through which the traveltimes received at point D is the minimum. This statement is expressed mathematically as:

$$dt/dz_0 = 0 \quad (3.5)$$

where t is given by equation (3.1). Substituting S_a^2 in equation (3.4) with equation (3.3), then taking the square root of equation (3.4), yields

$$t_0 = \left[(z_0^2 - z_1^2) \cdot (t_2^2 - t_1^2) / (z_2^2 - z_1^2) + t_1^2 \right]^{\frac{1}{2}} \quad (3.6)$$

This expression for t_0 is then substituted into equation (3.1), yielding

$$t = \left[(z_0^2 - z_1^2) \cdot (t_2^2 - t_1^2) / (z_2^2 - z_1^2) + t_1^2 \right]^{\frac{1}{2}} + S \cdot [(z_2 - z_0)^2 + \Delta x^2]^{\frac{1}{2}} \quad (3.7)$$

Equation (3.7) gives the traveltime t to the point D as the function of z_0 , the z component of the intersection of the incident ray and the left side of the cell. Taking the derivative of equation (3.7) for t with respect to z_0 in order to determine the Fermat criterion according to (3.5) and equating to zero yields

$$[z_0/t_0] \cdot [(t_2^2 - t_1^2) / (z_2^2 - z_1^2)] - S \cdot (z_2 - z_0) \cdot [(z_2 - z_0)^2 + \Delta x^2]^{-\frac{1}{2}} = 0 \quad (3.8)$$

The minimum traveltime t may be obtained from equation (3.7) after equation (3.8) is solved numerically for the value of z_0 by a common bisection method in which the solution to a function is obtained using the sign change of the function between a series of bisecting points within the solution range (see section 9.1 of Press et al, 1988).

3.2.2 Calculation Algorithm

Note that a single solution to equation (3.7) yields only a local minimum arrival traveltime because the energy received at the node arrives only from one possible direction. For example, the seismic energy received at node D arrives via the left-hand edge connecting nodes A and B shown in Figure 3.2; this traveltime may not be the global minimum which may come from a different direction through another boundary. In order to obtain the global

minimum arrival traveltime, i.e. the first arrival traveltime which will be assigned to point D, all possible directions from which energy is received at this point must be accounted for. This procedure to obtain the first arrival traveltime at a certain point is summarized in the calculation algorithm below. The algorithm essentially calculates all possible traveltimes to a given node from all directions and the minimum value is defined to be the first arriving traveltime by Fermat's principle.

Step 1. Initialization: It was noted from the last section that to calculate the traveltime to node D (Figure 3.2), the traveltimes at nodes A and B must already be known. Initialization is the process whereby a first set of calculated traveltimes are assigned to the nodes at which traveltimes are required to be known for any further calculation. For the present calculation geometry (Figure 3.1) the traveltimes to the nodes represented by open circles which share the same x -component as the source are assigned with the direct arrival traveltimes, which are simply calculated by the distance between the source and the node multiplied by the average slowness. Note that these initial values need not necessarily be the final first arrival traveltimes to these positions. Later as the calculations proceed these traveltimes may be replaced by the true first arrival traveltimes. Once this initialization of the traveltimes is completed, the calculation of traveltimes to the remainder of the nodes in the medium is carried out commencing in the adjacent column; in Figure 3.1 the calculations of the traveltimes proceed to the nodes denoted as solid circles. The calculation procedure is shown schematically in Figure 3.3.

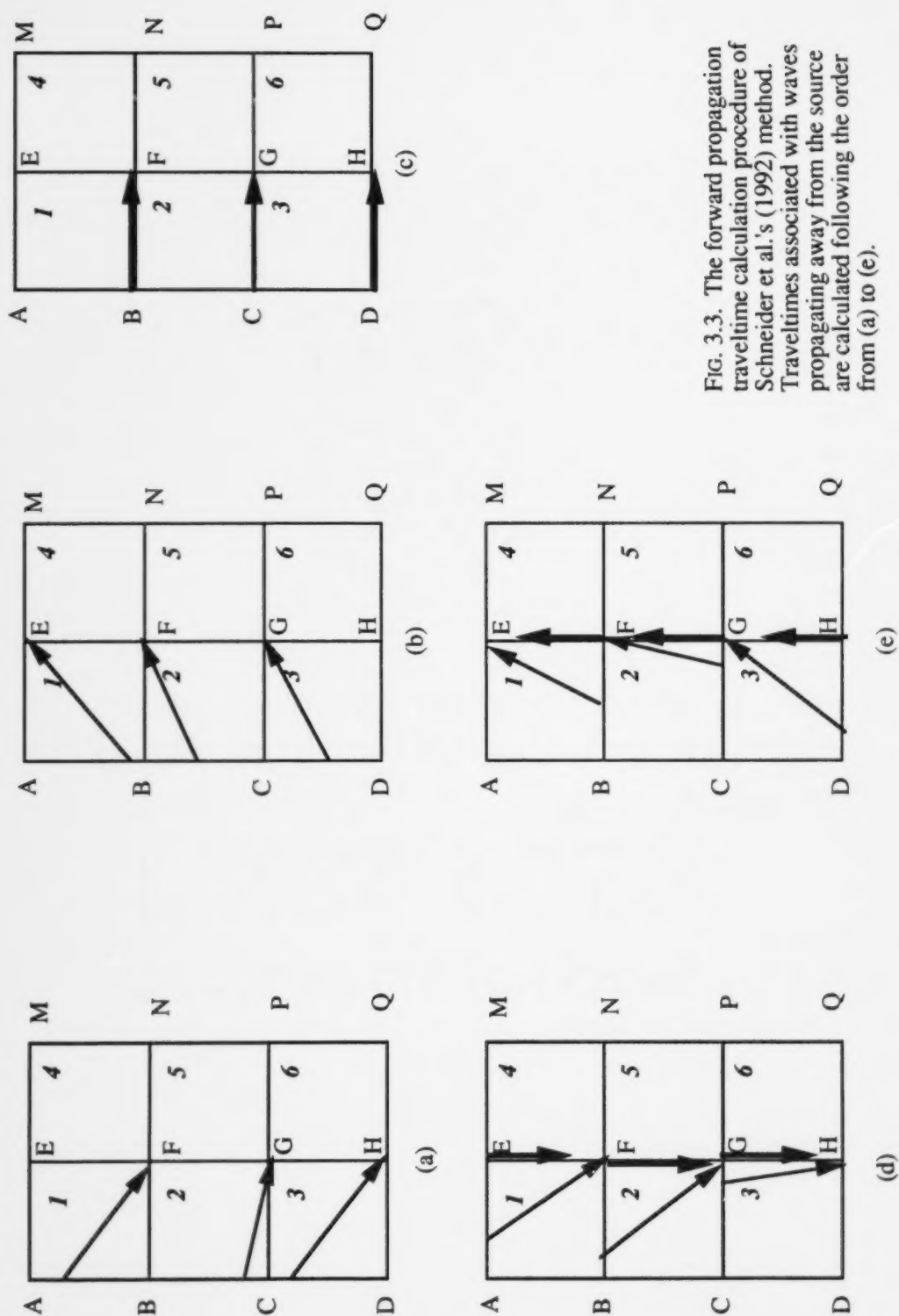


FIG. 3.3. The forward propagation traveltime calculation procedure of Schneider et al.'s (1992) method. Traveltimes associated with waves propagating away from the source are calculated following the order from (a) to (e).

Step 2. Horizontal propagating waves away from the source;

(a) Commencing with cell 1 in Figure 3.3 (a) which has already been initialized at nodes A through D, the traveltime to node F (lower cross corner of cell 1) is obtained from the known initialized traveltimes t_1 at node A and t_2 at node B by solving equations (3.7) and (3.8) as described. This calculation procedure is repeated for cell 2 whereby the traveltime to node G is obtained from the known traveltimes t_2 and t_3 at nodes B and C, respectively. Finally, the traveltime to node H is obtained from the known traveltimes t_3 and t_4 at nodes C and D for cell 3.

(b) This step is similar to step 2 (a), but now the traveltimes to the upper right hand corner nodes of each cell are calculated (Figure 3.3 (b)). The procedure starts at the bottom cell 3 and the traveltime to node G (upper cross corner of the cell) is calculated from the known traveltimes t_4 at node D and t_3 at node C by solving equations (3.7) and (3.8). This new value obtained is then compared with the previous value at node G calculated in step 2 (a); the smaller one is defined as the traveltime to node G. This calculation and comparison procedure is then repeated for cell 2 and cell 1 to provide the minimal traveltimes to nodes F and E, respectively.

Step 3. Horizontal refracted waves propagating away from the source (Figure 3.3 (c)): For a geologically realistic model, critically refracted waves exist and they are often referred to as head waves (Aki et al., 1980). In optics and seismology a ray is said to be critically refracted if the incident ray is contained in the slow velocity region and the incident angle of the ray is greater than the critical angle defined as $\theta = \sin^{-1}(v_{slow}/v_{fast})$, where v_{slow} and v_{fast} are the velocities on either side of the interface. The incident angle is

defined as the angle between an incident ray and the normal direction of the interface at the incident point. These critical refractions can significantly complicate a given set of seismic arrivals if not accounted for. The traveltimes associated with these possible head waves propagating along a cell boundary are really a special case to those of step 2 and can be calculated using the concept of a head wave. In a similar manner, these traveltimes are compared with those produced and stored in step 2, and the smaller value is defined as the traveltime to the node. For example, the traveltime to node F which is associated with the head waves propagating along segment BF (the boundary between cell 1 and cell 2) is calculated by equation

$$t_F = t_B + \Delta x \cdot S_{min} \quad (3.9)$$

where S_{min} is the smaller slowness (i.e., corresponds to the medium with the higher velocity) of the two slownesses between cells 1 and 2, and t_B is the traveltime at node B. t_F is then compared with that for node F stored after step 2, and the smaller one is assigned as the new traveltime to node F.

Step 4. Downward vertically and obliquely propagating waves away from the source: Moving back to cell 1 (Figure 3.3 (d)), the traveltime to node F is the solution to equations (3.10) and (3.11) below which are derived after switching x and z components in equations (3.7) and (3.8), respectively

$$t = [S_a^2 \cdot (x_0^2 - x_1^2) + t_1^2]^{\frac{1}{2}} + S \cdot [(x_2 - x_0)^2 + \Delta z^2]^{\frac{1}{2}} \quad (3.10)$$

$$x_0 \cdot S_a^2 / t_0 - S \cdot (x_2 - x_0) \cdot [(x_2 - x_0)^2 + \Delta z^2]^{-\frac{1}{2}} = 0 \quad (3.11)$$

The traveltimes t_1 at node A and t_5 at node E are used as the known times in solving these equations. The traveltime associated with possible head waves vertically propagating down along the boundary between cells 1 and 4 is

evaluated in a similar manner to that in step 3. Again after the comparison, the lowest value is defined to be the new traveltime. This calculation procedure is repeated for cell 2 and cell 3 where the new traveltimes to node G and H are defined, respectively, as illustrated in Figure 3.3 (d)

Step 5. Upward vertically and obliquely propagating waves away from the source: Step 4 gives solutions of the traveltimes associated with the downward propagating waves. In this step, the calculations are similar but proceed upward with traveltimes to the upper right hand nodes of each cell being calculated (Figure 3.3 (e)). The calculations are carried out from cell 3 to 1 whereby an updated least traveltime is assigned to nodes G, F and E, respectively.

Step 6. Progression to next column: This step actually repeats steps 2 to 5 but now the calculations are carried out on the column of cells 4 to 6. The traveltimes to nodes M, N, P and Q need to be defined from the now known (i.e., initialized) traveltimes at nodes E, F, G and H. This configuration then proceeds to the next right column (not shown in Figure 3.3) and continues until the traveltimes to the nodes in the right-most column are defined.

The above six-step procedure can be called the *forward propagation calculation*, and it includes only those waves propagating away from the source. In the co-ordinate system shown in Figure 3.1, this means that these waves satisfy the requirement $\nabla_x t > 0$ after completion of the above six steps. Here ∇_x is the x -component gradient operator defined as $\nabla_x = \hat{x} \cdot \nabla = \frac{\partial}{\partial x} \hat{x}$, where $\hat{x}$ is a unit vector in the x direction. In order to also include those waves propagating backwards toward the source (i.e. those for which $\nabla_x t < 0$), a set of similar but direction reversed calculations or *backward propagation*

calculations are run through all the cells. In this procedure, traveltimes are obtained progressively towards the left, i.e., traveltimes to the left hand nodes of cells are calculated from the ones at the right hand nodes of the cells. The reverse run uses the already obtained traveltimes at the nodes on the most-right hand column as the initial values and also only exchanges traveltimes with the lowest value at each node during the calculations. As the least arrival traveltime is defined to each node, this two pass algorithm (forward and backward) enables one to obtain the first arrival traveltimes to nodes in any media with arbitrary velocity distribution. One advantage is that the first arrival traveltimes obtained are associated with either a refracted wave, a head wave or a diffracted wave. As such, all three of the arrival types are automatically accounted for.

It should be noted here that the cell geometry shown in Figure 3.1 is used only for its simplicity in assisting with the explanation. Though a realistic geological model will be much more complicated, the basic calculation principle listed in this section will not fundamentally change. The computational flow chart of Schneider et al.'s (1992) method is shown in Figure 3.4 and the C-code of this method is provided in the Appendix.

3.2.3 Numerical example

Figure 3.5 illustrates a high contrast, discontinuous velocity model. A low velocity block is embedded in a homogeneous high velocity region; the velocities are 2080 m/s and 3300 m/s, respectively. The horizontal and vertical dimensions of the low velocity block and the model structure are 100 m and 150 m, 400 m and 300 m, respectively. The model contains 80 and 60 square cells in the horizontal and vertical directions, respectively, these cells

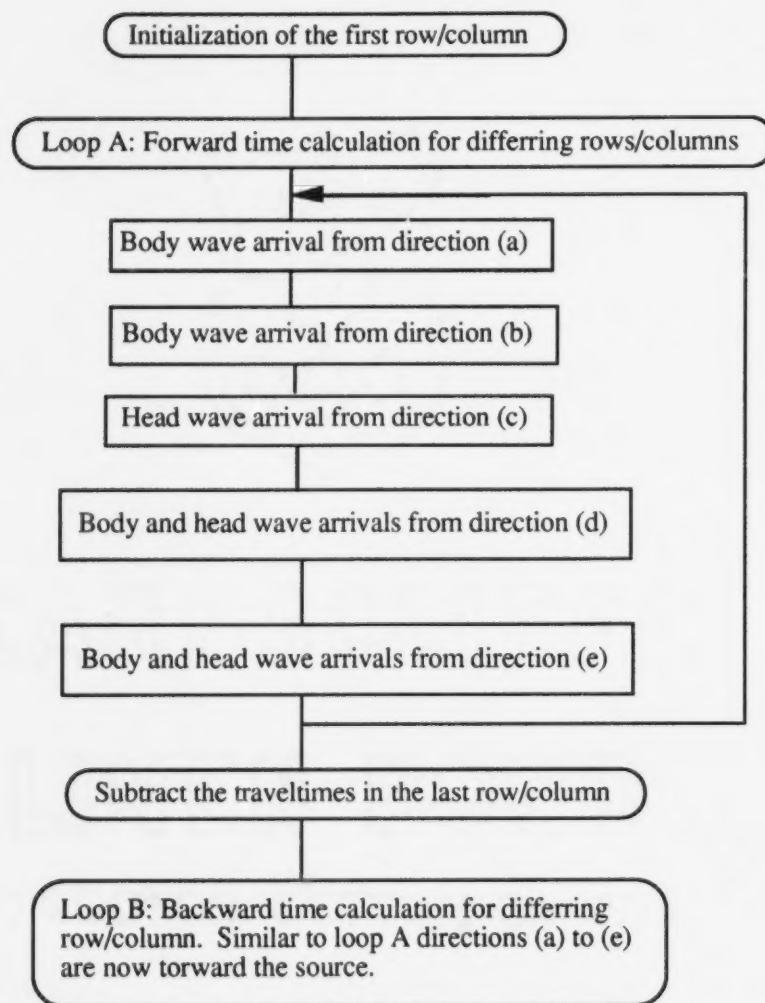


FIG. 3.4. The computational flow chart of Schneider et al.'s method. Directions (a) to (e) are referred to figure 3.3.

are 5 m in sides of length. The location of the block is illustrated in Figure 3.5 and the source is located at the upper left-hand corner of the model. Note that in Figure 3.5 the horizontal and vertical scales are different.

The first arrival traveltimes are obtained by the use of above calculation scheme and the contoured traveltimes with the time intervals of 7.9 milliseconds are illustrated in Figure 3.6. Note that the iso-time lines also represent the leading wavefronts. This model shows a variety of wave types, including diffractions (e.g., the region adjacent to the upper right-hand and lower left-hand corners of the block) and complicated headwaves (e.g., along the right vertical side and the bottom horizontal edge of the block). Note that the first arriving energy has been diffracted around the block to reach the lower right-hand corner of the model whilst the energy traveling directly through the block would appear at this corner as a later arrival. However, it must be noted that the propagating energy discussed here is only in a sense of a qualitative but not a quantitative property. The first arrival wavefronts often tend to have the maximum geometrical spreading effect. Consequently, the wave energy associated with the first arrival can be very small and difficult to distinguish in some complex geological structures (Geoltrain et al., 1993).

The computation of traveltimes in Figure 3.6 required 15 seconds of CPU time on a SPARC-2 SUN Station. At present, we have not compared this time to that which would be required for other techniques but the results indicate that it would be useful for many of the advanced tomographic imaging methods now being utilized. Schneider et al. (1992) also pointed out that the errors of this method are only numerical round-off errors and are within 0.2 percent, the magnitude of the error depends on the model studied.

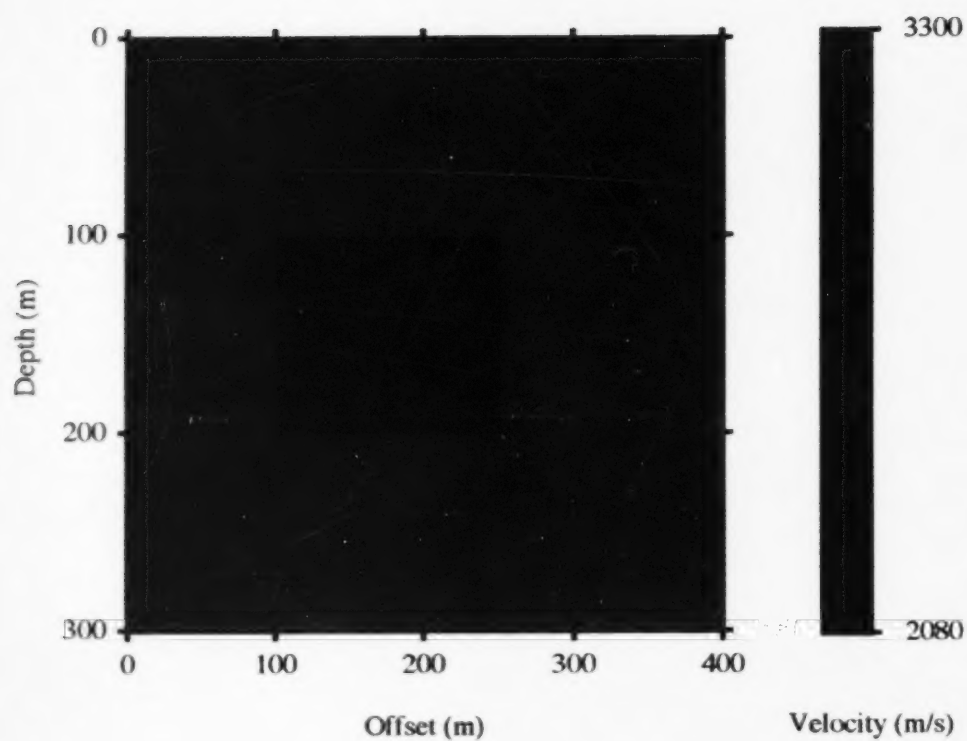


FIG. 3.5. Block velocity model. The velocities are 2080 m/s and 3300 m/s within and outside the square, respectively. Different scales are used in horizontal and vertical directions.

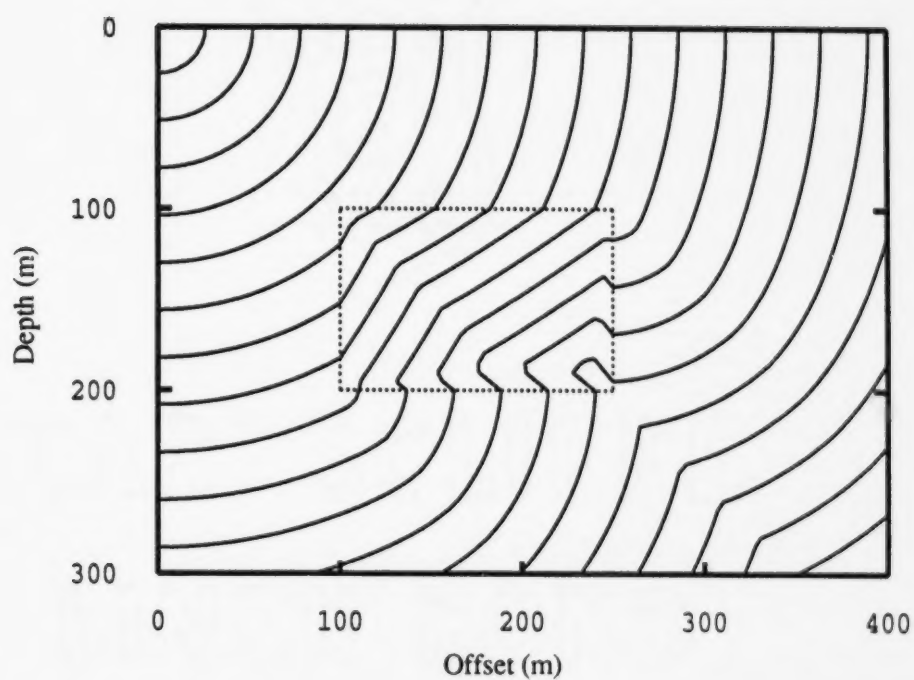


FIG. 3.6. First arrival traveltimes contours for the model shown in figure 3.5. The time intervals between the contours are 7.7 ms. The block edges are represented by the dashed lines.

3.3 Reflected Traveltime Calculation — A Modified Schneider et al.'s Method

Schneider et al.'s (1992) method remains accurate in obtaining the first arrival traveltimes even in situations where there is a large contrast in seismic velocities between adjacent grid cells. Both conventional ray tracing and finite difference methods have difficulty with rapid changes in material velocity. However, traveltimes associated with reflections can not be handled. This problem is not unique to Schneider et al.'s (1992) method, and other traveltime calculation methods mentioned earlier also have the same difficulty (e.g., Vidale, 1988, 1990; Podvin et al., 1991; Moser et al., 1991; Van Trier et al., 1991; Qin et al., 1992; Asakawa et al., 1993, and Coultrip, 1993). As noted, Matsuoka et al. (1992) attempt to offer an alternative reflection traveltime calculation which essentially relies on the principle of reciprocity. However, their method is computationally time consuming and, because it based on Vidale's (1980, 1990) method, the potential causality problem can not be avoided. In this section, a new calculation method for calculating reflected traveltimes is developed. This method employs the essential components of Schneider et al.'s (1992) technique. Below is discussed an attempt at developing a method for calculating reflection time. This technique is first developed for a specific horizontal and flat reflector, then modified to be applied for the more general and useful cases of dipping and irregular reflectors.

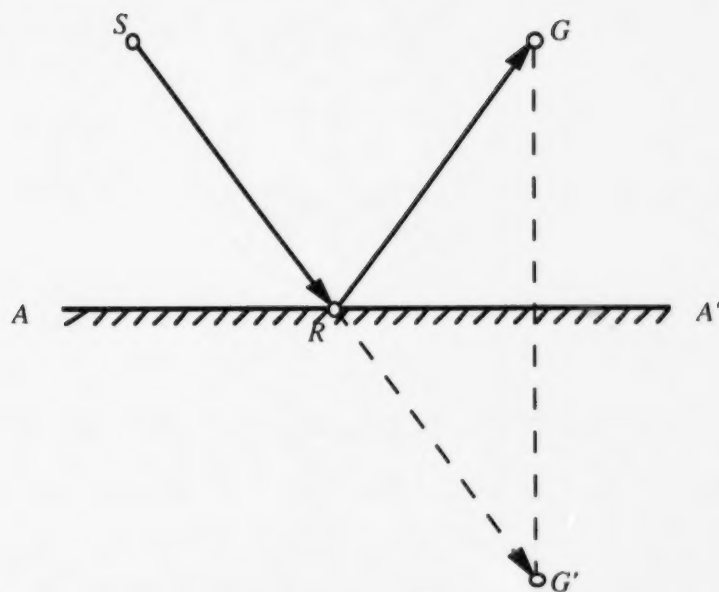


FIG. 3.7. Reflected rays observed at point G can be treated as if they were transmitted rays which were received at its virtual image point G'.

3.3.1 Reflected Traveltime Calculation via Virtual Image and Virtual Receiver

Although the geological structure is a very simple flat layering reflector in this section, it is instructive to understanding the basic technique which is later used for more complex calculations of the traveltimes associated with reflected waves. From basic geometric optics, it is well known that reflected rays (Figure 3.7) received at a receiver G (an arbitrary point) can be treated as if they were transmitted rays which were received at its virtual image point G'. This statement holds regardless of how complex is the medium, provided a virtual receiver within a virtual medium can be found. For example, the virtual medium is obtained by folding down the physical medium along the reflector AA' in Figure 3.7. It is also well known that since an incident and its associated reflected wavefield are assumed not to interact with each other, it is valid to calculate the incident and the reflected wavefield separately. With the above two basic assumptions dictating the behavior of the wave reflections, Schneider et al.'s (1992) method can be modified to include the calculation of reflected traveltimes.

Figure 3.8 (a) illustrates a flat layered velocity model whereby the velocity as a function of the depth is illustrated in Figure 3.8 (b). There are 5 major layers in this model. The first layer is from the surface to a depth of 20 meters within which the velocity gradually increases from 1240 m/s to 1520 m/s. The second layer is located in a depth range between 20 to 70 meters. There is a 180 m/s velocity discontinuity between the first and the second layer, the velocity linearly increases from 1700 m/s to 2030 m/s in the second layer. After a 50 m/s change in the velocity, the velocity linearly increases from 2080 m/s to 2240 m/s in the third layer which is located in

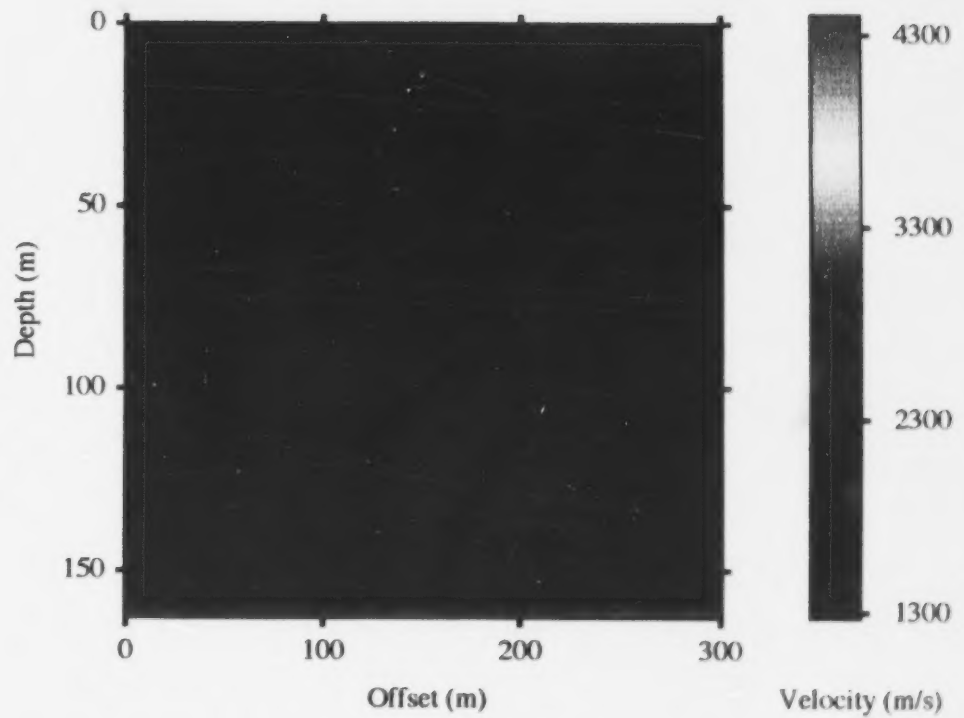


FIG. 3.8 (a). Horizontal layered velocity model. Velocities are coded by varying levels of colours. Different scales are used in horizontal and vertical directions.

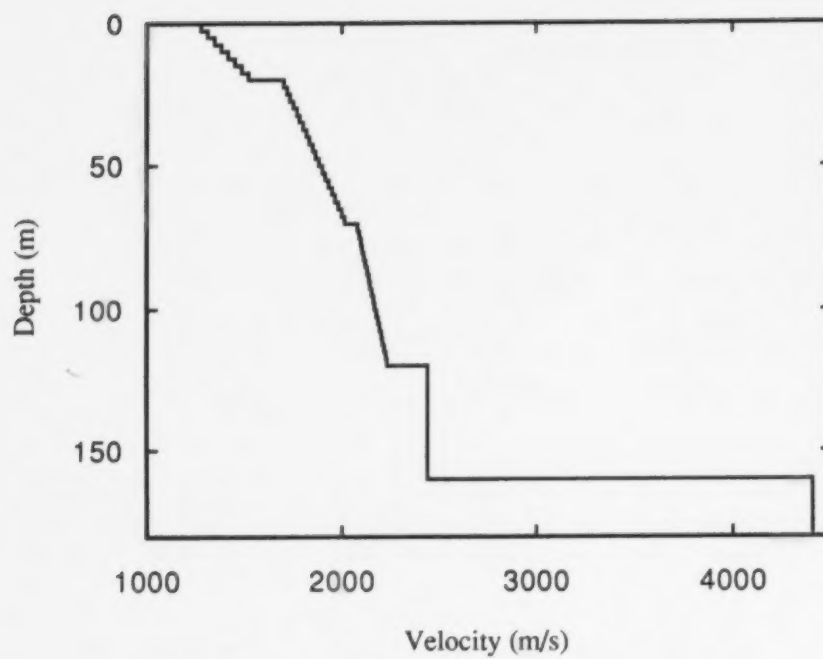


FIG. 3.8 (b). Layered velocity function. There are five major layers in the model.

the depth from 70 to 120 meters. The last two are velocity-constant layers with the velocities of 2440 m/s and 4400 m/s, respectively. The bottom layer (i.e., velocity 4400 m/s) is assumed to be a carbonate layer and a strong reflection from its top interface is expected.

In order to obtain the reflected arrivals, a virtual imaging of the top 4 layers (i.e., the medium above the reflector at the depth of 160 meters) is created by folding the velocity structure down along the top interface of the carbonate layer (Figure 3.9 (a)). The velocity function associated with this new virtual medium is illustrated in Figure 3.9 (b). The first arrival traveltime field is then obtained for this new model (Figure 3.9) using Schneider et al.'s (1992) method. The first arrival traveltime fields obtained in the top and the bottom half (i.e., in the real and virtual medium) correspond to the wavefronts that are incident to and reflected from the carbonate, respectively. The numerical calculation results of the first arrival traveltime field, the reflected traveltime field (including incident traveltime field) and the reflected traveltimes observed at the surface on the given velocity model (Figure 3.8) are shown in Figure 3.10, Figure 3.11 (a) and (b), respectively. The layer interfaces are marked as the light horizontal dashed lines in these two figures. The time intervals between the contours are 11 milliseconds. One note is that only the reflections from the top of the carbonate layer have been accounted for. An advantage of this method is that the traveltimes corresponding to the converted reflected waves could also easily be calculated by exchanging the P for the S wave velocities during the calculations through the lower virtual medium.

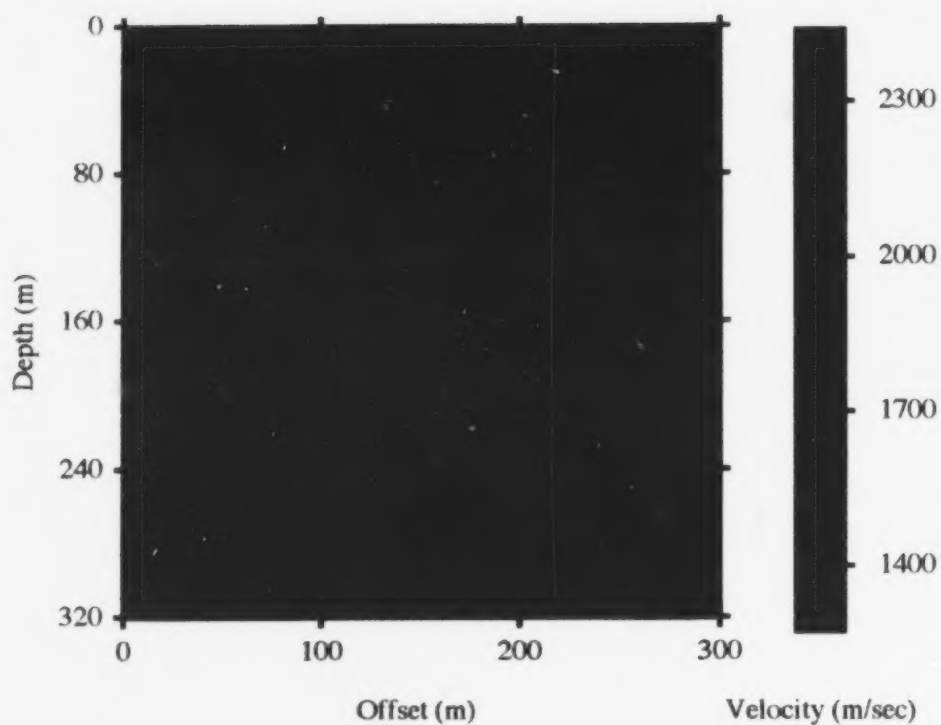


FIG. 3.9 (a). Velocity model from the virtual image of the model in figure 3.8 (a).

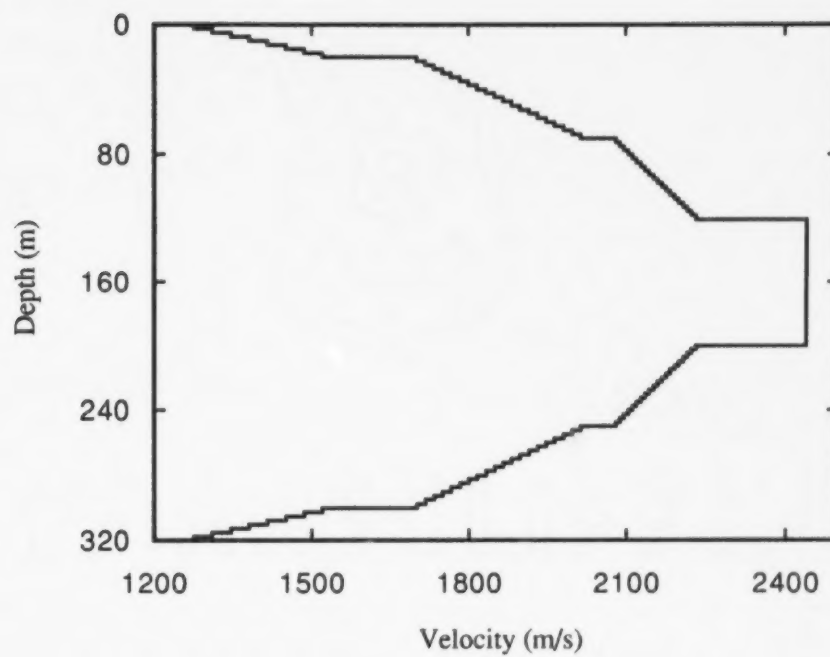


FIG. 3.9 (b). Layered velocity function after the virtual image of the model shown in figure 3.8 (b)

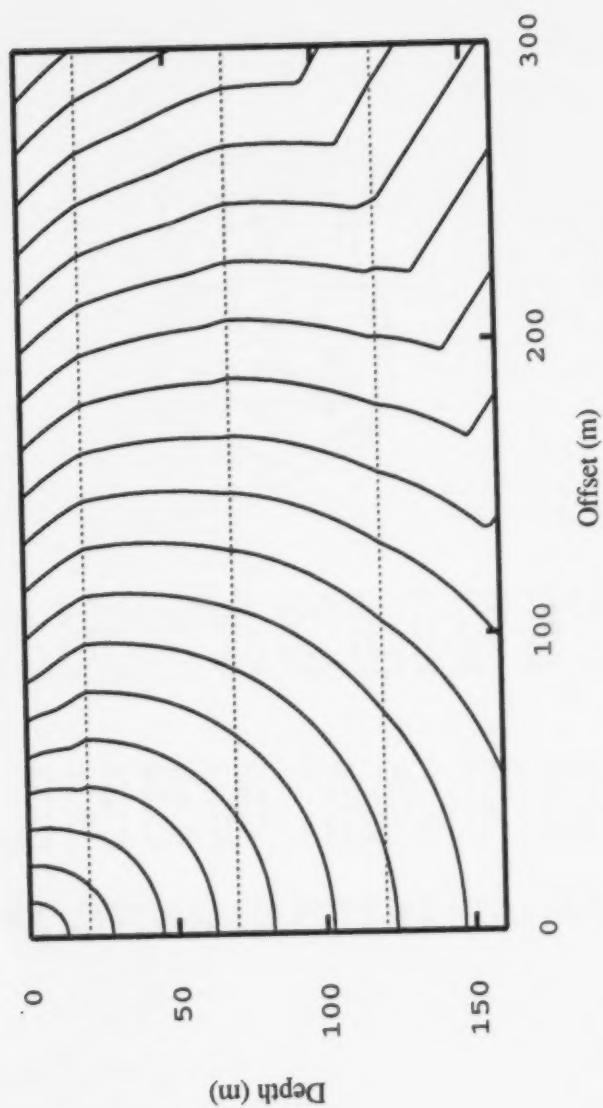


FIG. 3.10. First arrival traveltimes contours of the model shown in figure 3.8. The time intervals between the contours are 9.4 ms. The dashed lines represent the major interfaces.

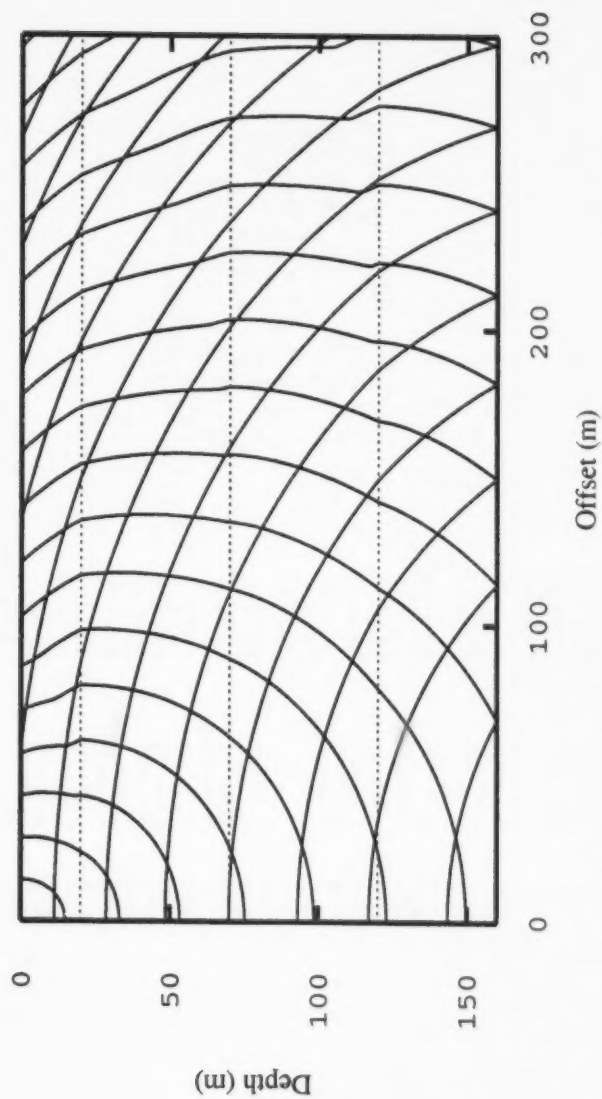


FIG. 3.11 (a). Traveltime contours of the incident and reflected waves for the model shown in figure 3.8. The time intervals between the contours are 11 ms. The dashed lines represent the major interfaces.

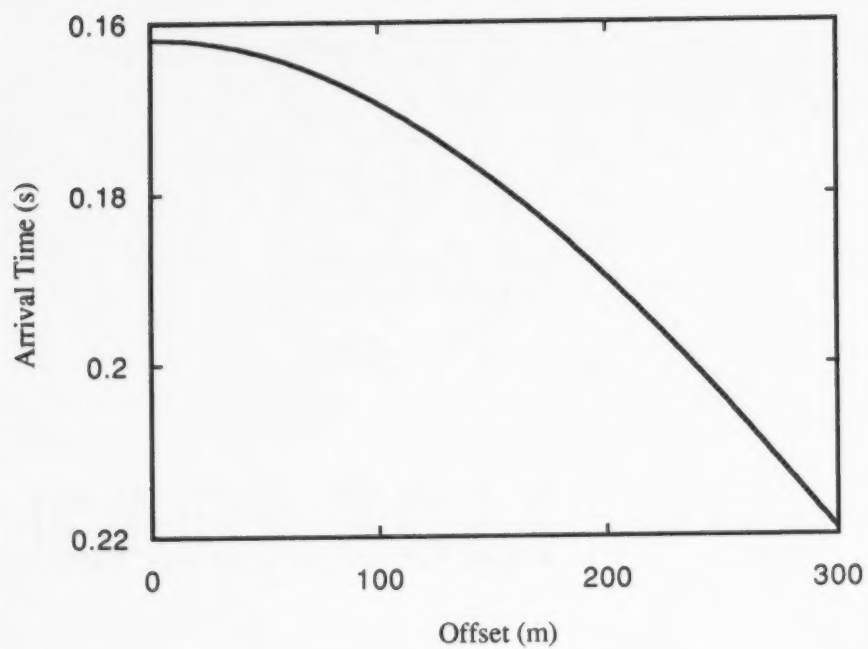


FIG. 3.11 (b). Arrival traveltimes reflected from the carbonate and observed at the surface for the model shown in figure 3.8

3.3.2 Reflected Traveltime Calculation in a Complex Geological Structure

In a geological realistic structure, there are always multiple interfaces in a medium and the shapes of these interfaces are complex. This latter fact makes it difficult or impossible to create a virtual image. This difficulty can be remedied by changing the calculation scheme noted in section 3.3.1.

As noted in the earlier sections, Schneider et al.'s (1992) calculation algorithm proceeds by columns (or rows) of cells; it is also instructive to analyze the above reflection calculation via the virtual medium technique from a different angle. The reflection calculation algorithm presented in the last section can be reconstructed by two major steps:

Step 1. The first arrival traveltime calculation algorithm proceeds in the real medium (the top half in Figure 3.9 (a)) and yields the traveltimes associated with the waves incident to the reflector. The purpose of this calculation is to provide a set of initialized traveltimes (the true arrivals for the incident wave) along the reflecting interface.

Step 2. Constrained by these traveltimes along the reflecting interface, Schneider et al.'s (1992) calculation algorithm then proceeds again but in the virtual medium only (the bottom half in Figure 3.9 (a)) to yield the expected traveltimes associated with reflected waves.

As the source is located on the top of the structure, the calculations progress commencing in the top row both in step 1 for the real and step 2 for the virtual media. However, an important change is now made in step 2. Constrained by the traveltimes along the reflecting interface, the calculation algorithm proceeds *back into the real medium via the backward and then*

forward propagation procedure. That is, the calculations commence in the lowest row in the real medium which contains the reflecting interface. Because of this small but important change, the virtual medium is no longer explicitly needed and the difficulty of finding the virtual medium is eliminated.

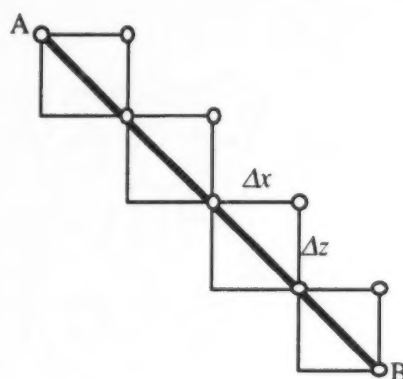


FIG. 3.12. A dipping reflector is modeled as a series of steps represented by the open circles.

It is natural to raise the question regarding the technique used to model an arbitrary interface between two geological media of differing velocity. Coultrip (1993) provides a calculation scheme which uses triangle cells in order to obtain a reasonable matching between the cell blocks and the velocity structure when modeling interfaces. Li et al. (1993) use a combination of a set of small rectangular and triangle subcells along interfaces to reduce the mismatching error. Here we use only rectangular cells when boundary matching; the horizontal and vertical dimensions of a cell on an interface are chosen as such that a segment of the interface is always on the line connecting the two opposite corners of the cell. For example, as illustrated in Figure 3.12, a dipping interface segment AB is now approximately represented by downward discrete steps which trace along the open circles. An analytic

estimation of errors caused by the boundary mismatching of this technique will be discussed in the coming section.

Based on the modified two-step calculation procedure and boundary matching technique, the reflection traveltimes calculation algorithm in a medium with a dipping or a more complex interface is provided as follows:

Step 1. The dimensions of the horizontal and vertical sides of the cells astride the reflecting interface are chosen so that the above boundary matching requirement is satisfied. These special cell dimensions are only used in the region surrounding the reflecting interface and the sizes of the cells in the rest region of a model are chosen based on the calculation accuracy requirements.

Step 2. The forward and backward propagation calculations noted earlier in this section are carried out on the cells above the reflecting interface, commencing in the row where the source is contained as noted above. To eliminate any potential headwaves (i.e., no head waves are included for the incident waves along the reflecting interface) and to simplify the numerical calculation, the velocity immediately below the reflecting interface is replaced by that of immediately above the interface. For example, the velocity in cell 9 (Figure 3.13) is replaced with that in cell 6 in order to ignore the possible head waves in the calculation. The traveltimes associated with the incident waves to the reflecting interface are obtained in this step.

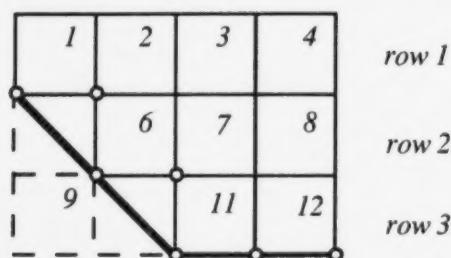


FIG. 3.13. A simplified geometry illustrated the new reflection traveltime calculation method. Commencing in row 1, the incident traveltime field is calculated using Schneider et al.'s method. Then constrained by the arrival times at the open circles, the reflected traveltime field is obtained.

Step 3. The backward and then forward propagation calculations noted earlier in this section proceed in the same region as step 2, commencing in the row where the lowest segment of the reflecting interface is contained. For example, the lowest row calculated in step 2 is the one which contains the three nodes represented by the open circles in the bottom of cells 11 and 12 in Figure 3.13. The traveltimes in this row obtained in step 2 represent traveltimes for both the incident and reflected waves on the reflecting interface segment and are used as initial values for step 3. During the coming two direction calculation procedure (i.e., the backward and the forward), the traveltimes at the nodes on the remainder of reflecting interface segments are considered as constraining conditions which are used to adjust the calculated traveltimes in the reflected region. For example, traveltimes at four nodes represented by the open circles at the bottom two corners of cells 1 and 6 in Figure 3.13 are used to modify the traveltimes in the rest of the region calculated via the normal procedure. This technique is based on the principle that each point on the reflecting interface acts as a Huygens source which is

activated at the arrival time to the node. This step yields the traveltimes associated with the reflected waves.

This three-step calculation procedure enables one to calculate reflected traveltimes with a cost of only twice the computing time that used for first arrival traveltime calculation in Schneider et al.'s (1992) method. The calculation error, which will be discussed in next section, of this method is primarily the result of the mismatching of the reflecting interface and is small. These two advantages make it practically useful in the fields related to the reflection traveltime (e.g., reflection traveltime tomography, migration etc.). Since any complex interfaces may be modeled by the combination of a series dipping and fault interfaces, this method can be used in the reflection traveltime calculation in a medium with any arbitrary interfaces. Three numerical examples are illustrated in next section, i.e., a dipping reflector, a fault reflector, and a syncline reflector, respectively.

3.3.3 Numerical Examples and Error Analysis

Three velocity models are used to demonstrate the reflection traveltime calculation method. These models all exhibit a high contrast, discontinuous velocity structure; the lower and the higher velocities are 2440 m/s and 4400 m/s, respectively. The Cartesian co-ordinate system is chosen such that x is in the horizontal and z is in the vertical directions, the origin is at the upper left hand corner in each of the following models.

(a). Fault

The first model is a fault (Figure 3.14 (a)). The fault occurs at the offset of $x=150$ meters whereby the source is located at the origin of the co-ordinate as illustrated in Figure 3.14 (a). The dimensions of the model are 400 and 350 meters in the horizontal and vertical directions, respectively; the dimensions of the cell used in the traveltimes calculations are 5 by 5 meters.

Figure 3.14 (b) illustrates the first arrival traveltimes contours (i.e., the leading wavefronts); the planar wavefronts occurring above the interface at the depth of $z=300$ meters shows the evidence of the headwave along the interface. Figure 3.14 (c) illustrates the traveltimes associated to the incident and the consequent reflected waves; a strong diffraction occurs at the upper corner (150, 150) in the model and the shadow zone is formed below the diffracting corner. The "reflections" received at the surface ($z=0$) (Figure 3.14 (d)) consist of two types of arrivals: the true reflections from the upper interface ($z=150$ meters) and the diffractions from the upper corner (150, 150) of the fault are observed in the surface regions of $0 < x < 300$ and $300 < x < 400$ meters, respectively (Figure 3.14 (d)). The relative percentage errors for the "reflected" traveltimes are illustrated in Figure 3.14 (e). The sector shaped error distributions indicate that the errors are primarily the result of the diffraction calculations. However with a maximum of 0.7 percent error, these results are considered accurate.

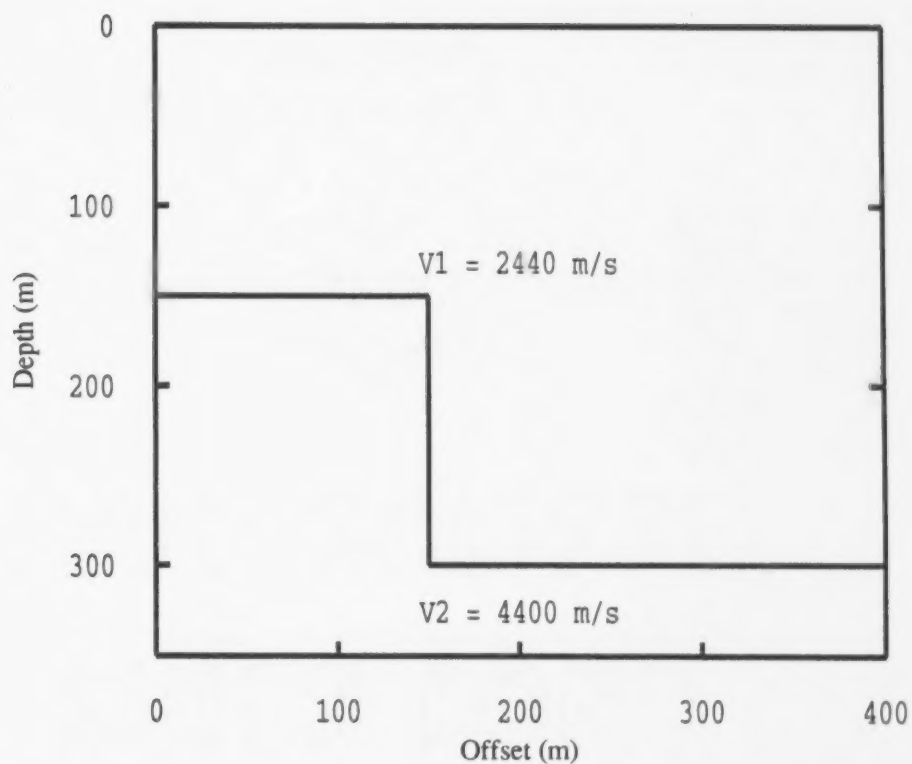


FIG. 3.14 (a). Velocity model for a fault structure. The solid lines represent the interfaces between the differing velocities. The source is located at (0,0) of the model.

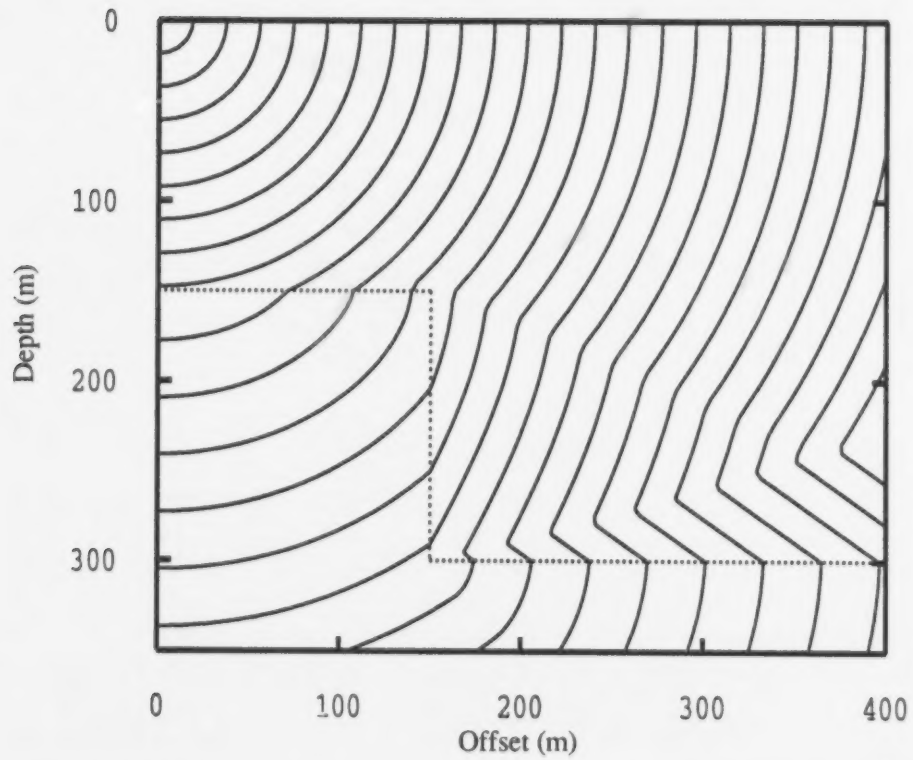


FIG. 3.14 (b). First arrival traveltimes contours for the fault structure shown in figure 3.14 (a). The time intervals between the contours are 7.6 ms. The dashed lines represent the interfaces.

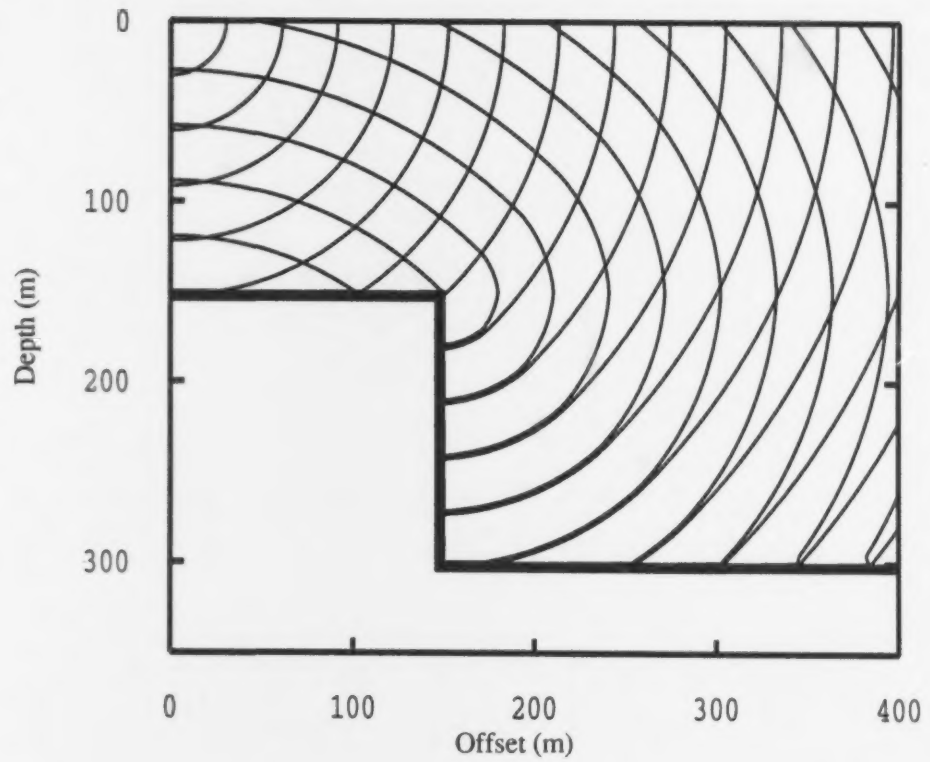


FIG. 3.14 (c). Traveltime contours of the incident and reflected (including diffracted) waves for the fault structure shown in figure 3.14 (a). The time intervals between the contours are 12.5 ms.

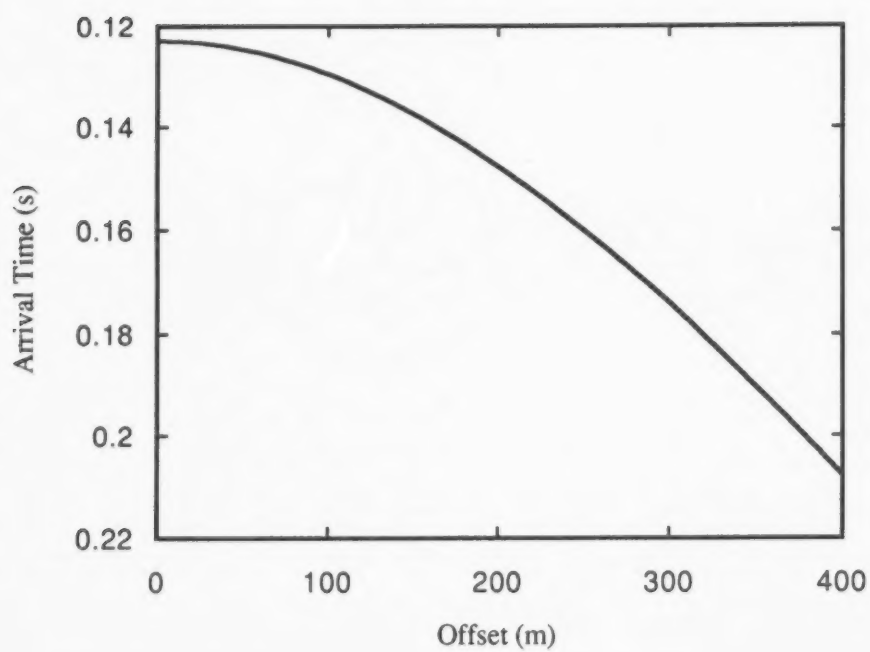


FIG. 3.14 (d). Arrival traveltimes reflected and diffracted from the interface shown in figure 3.14 (a) and observed at the surface.

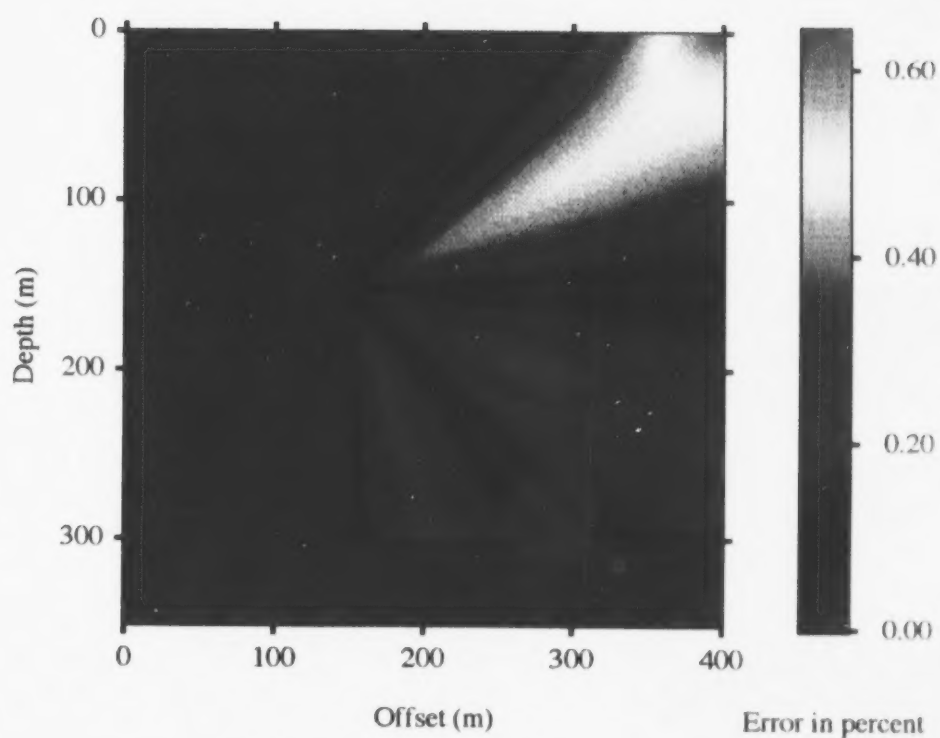


FIG. 3.14 (e). The relative percentage error map of the calculated reflection traveltimes for the fault structure shown in figure 3.14 (a). Errors are coded by varying levels of colours.

Figure 3.15 (a) illustrates a similar fault model. In this model, the fault occurs at the offset of $x=100$ meters whereby the source is located at the middle of the surface at (200, 0) meters. The model and cell dimensions are otherwise the same as those in Figures 3.14. The first arrival traveltimes, the incident and the reflected leading wavefronts and the "reflected arrival" traveltimes observed at the surface are illustrated in Figure 3.15 (b), (c) and (d), respectively. Figure 3.15 (b) illustrates that head waves first propagate along the vertical interface of the fault then turn to the lower horizontal interface (i.e., on the interface from (100, 300) to (150, 300)). The diffractions (Figure 3.15 (c)) from the upper corner (100, 150) of the fault dominate the "reflected" wavefronts; the leading wavefronts of the reflections from both the upper and lower horizontal interfaces are found only in the region closely above these interfaces (Figure 3.15 (c)). As a result, the "reflections" received at the surface (Figure 3.15 (d)) are diffracted wavefronts from the upper corner (100, 150) of the fault. The cusp in the traveltime curve in Figure 3.15 (d) is a source-related error. The error distribution for the "reflected wavefronts" is illustrated in Figure 3.15 (e). Again the primary errors are caused by the diffracted traveltime calculations.

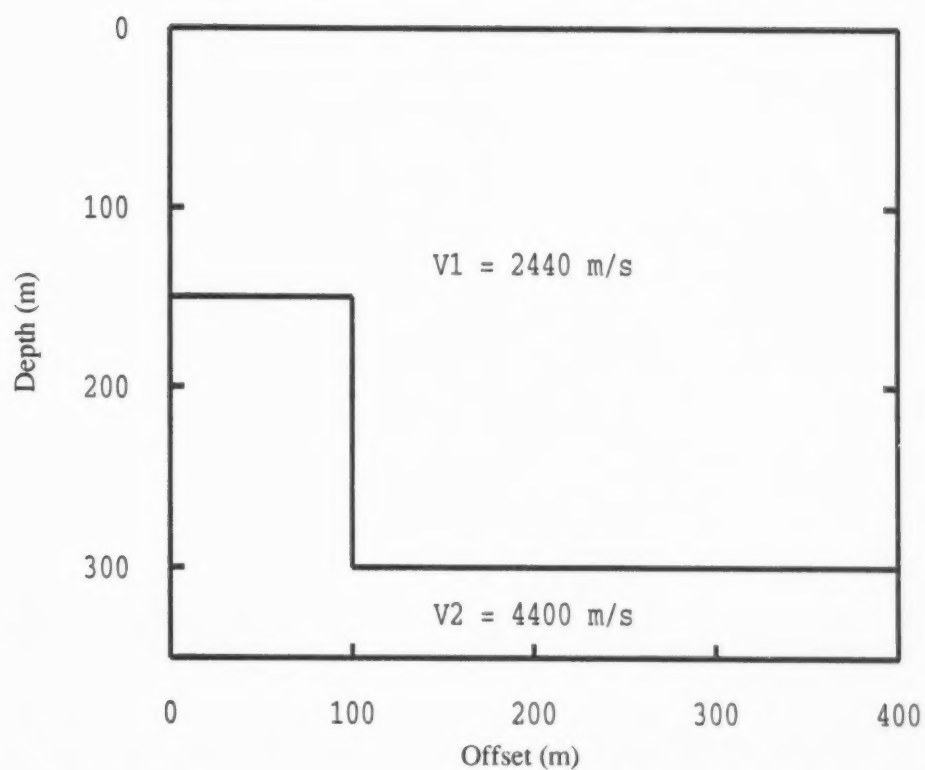


FIG. 3.15 (a). Velocity model for a fault structure. The solid lines represent the interfaces between the differing velocities. The source is located at (200,0) of the model.

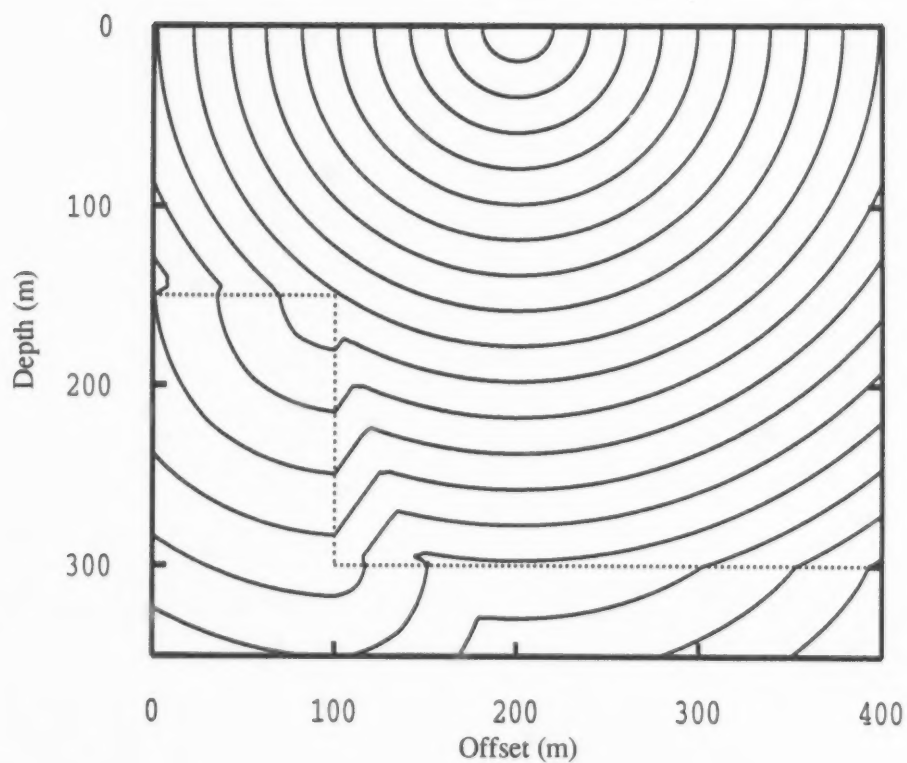


FIG. 3.15 (b). First arrival traveltimes for the fault structure shown in figure 3.15 (a). The time intervals between the contours are 8.2 ms. The dashed lines represent the interfaces.

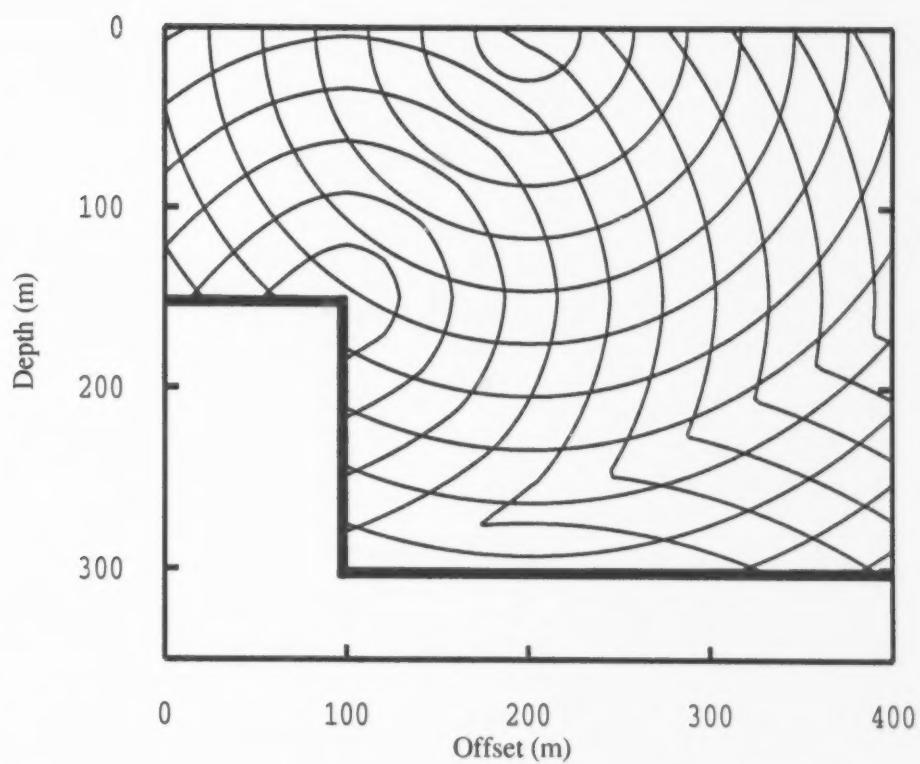


FIG. 3.15 (c). Traveltime contours of the incident and reflected (including diffracted) waves for the fault structure shown in figure 3.15 (a). The time intervals between the contours are 12.0 ms.

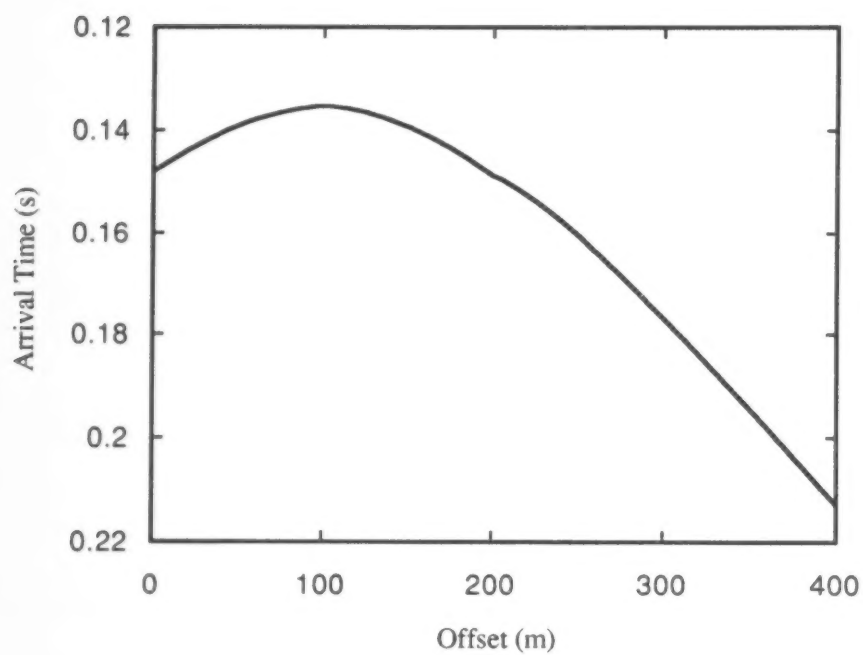


FIG. 3.15 (d). Arrival traveltimes reflected and diffracted from the interface shown in figure 3.15 (a) and observed at the surface.

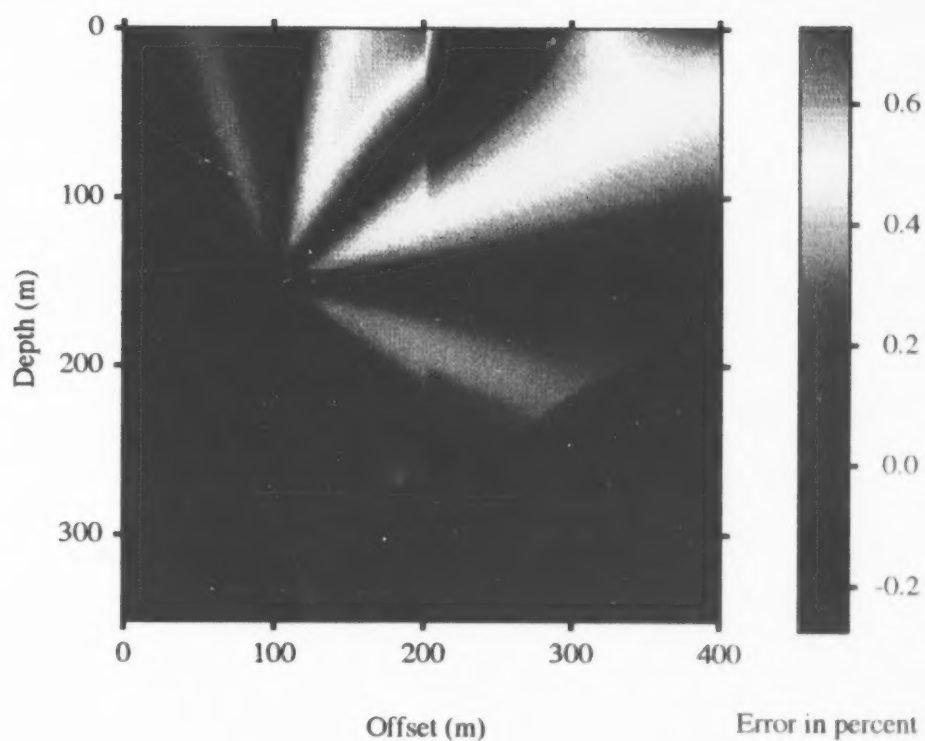


FIG. 3.15 (e). The relative percentage error map of the calculated reflection traveltimes for the fault structure shown in figure 3.15 (a). Errors are coded by varying levels of colours.

(b). Dipping Layer

The second model shows a dipping structure (Figure 3.16 (a)). The solid line in Figure 3.16 (a) illustrates the interface with a 18° dip angle which is modeled as a series of steps in the calculations. The source is located at the middle of the surface (i.e., (200, 0)). The dimensions of the model are 400 and 300 meters in the horizontal and vertical directions, respectively; the dimensions of the cells are 5 meters in both the horizontal and vertical sides above the depth of 165 meters; and 5.0 and 1.67 meters in the horizontal and the vertical below this depth, respectively. The dimensions of the cells are chosen based on requirement of the calculation algorithm noted in section 3.3.2. The contours of the first arrival traveltimes are illustrated in Figure 3.16 (b). The traveltimes associated to the incident and the reflected waves are shown in Figure 3.16 (c); the cusps of the reflected wavefronts immediately below the source in the figure are the effects of the model step reflecting interface. The locus curve of the traveltimes observed at the surface is a hyperbola as shown in Figure 3.16 (d). The errors (Figure 3.16 (e)) in the left hand region above the reflecting interface are primarily a result of the boundary mismatching. Alternatively, the lenticular shaped lobe of higher errors to the right are the result of the strong diffractions from the step corner immediately below the source. The maximum error is around 1.6 percent which can be reduced by a better type of the boundary matching. Note, however, that the errors in the traveltimes observed at the surface are all less than 1 percent.

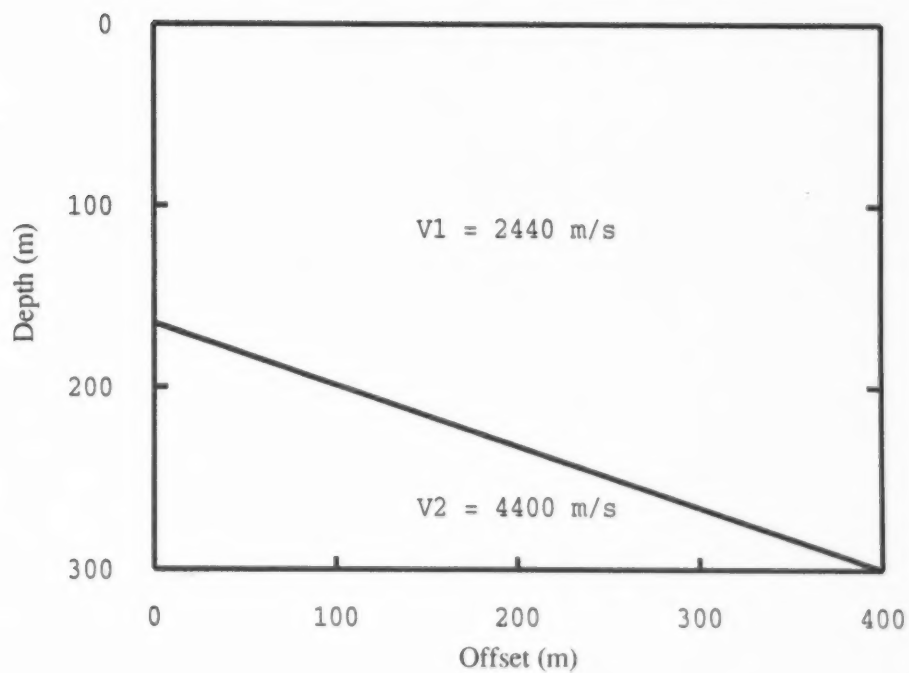


FIG. 3.16 (a). Velocity model for a dipping structure with a 18 degree dip angle. The solid line represents the interface between the differing velocities. This interface is modeled as a series of steps. The source is located at (200,0) of the model.

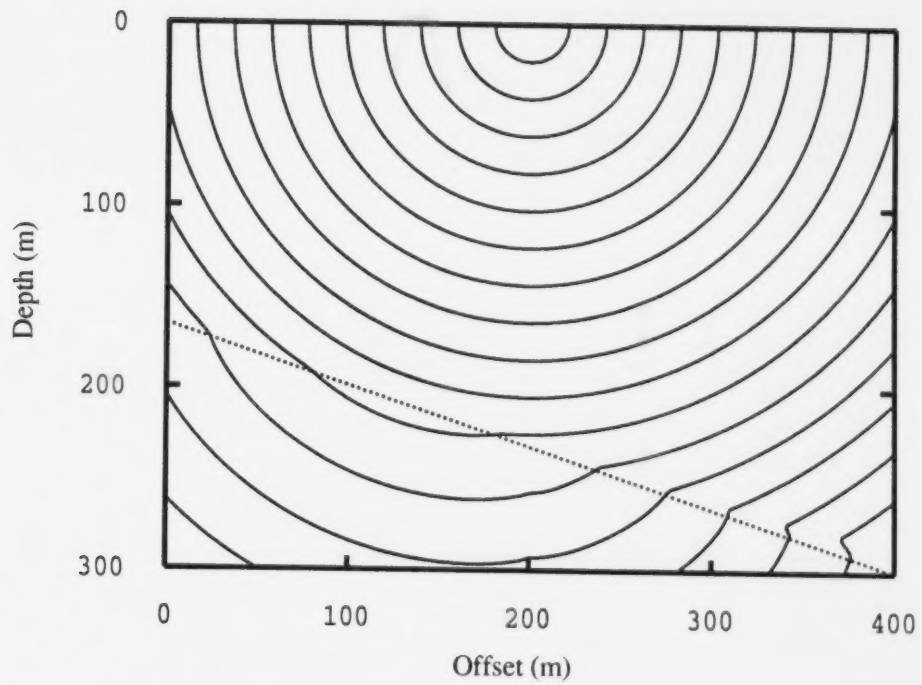


FIG. 3.16 (b). First arrival traveltime contours for the dipping structure shown in figure 3.16 (a). The time intervals between the contours are 8.4 ms. The dashed line represents the interface.

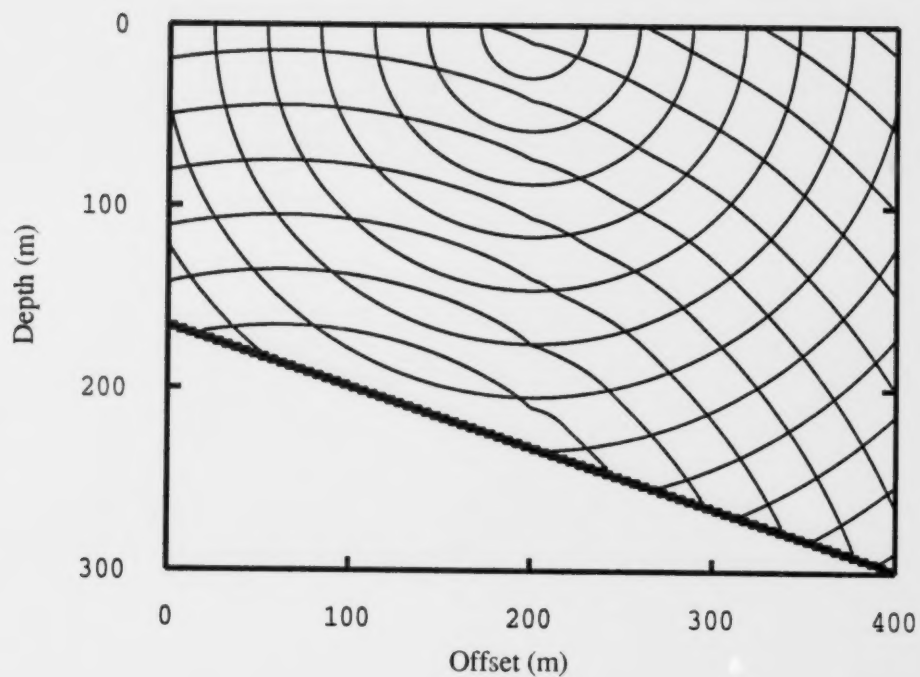


FIG. 3.16 (c). Traveltime contours of the incident and reflected waves for the dipping structure shown in figure 3.16 (a). The time intervals between the contours are 12.2 ms. The cusps of the reflected wavefronts are the step model effects.

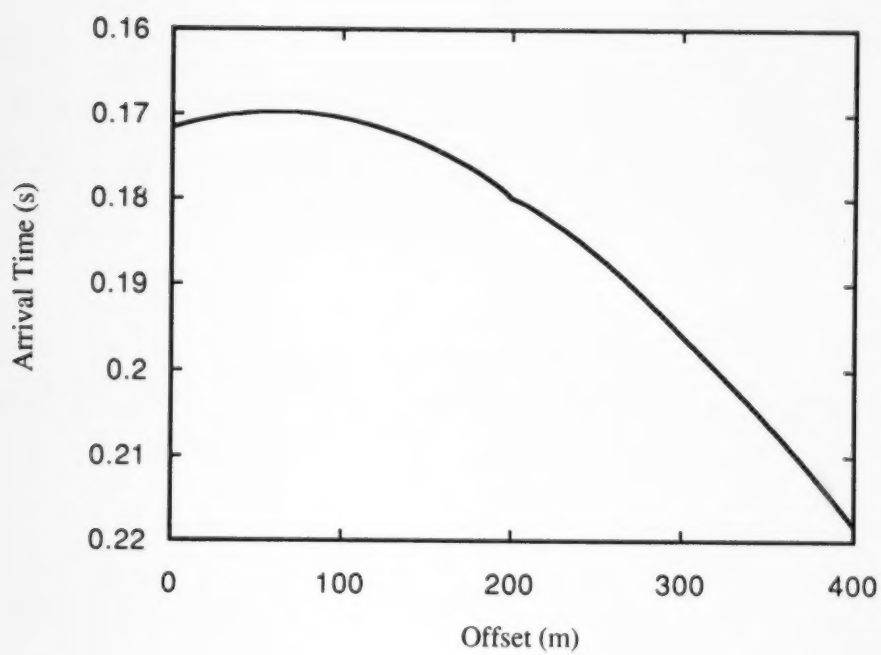


FIG. 3.16 (d). Arrival traveltimes reflected from the dipping interface shown in figure 3.16 (a) and observed at the surface.

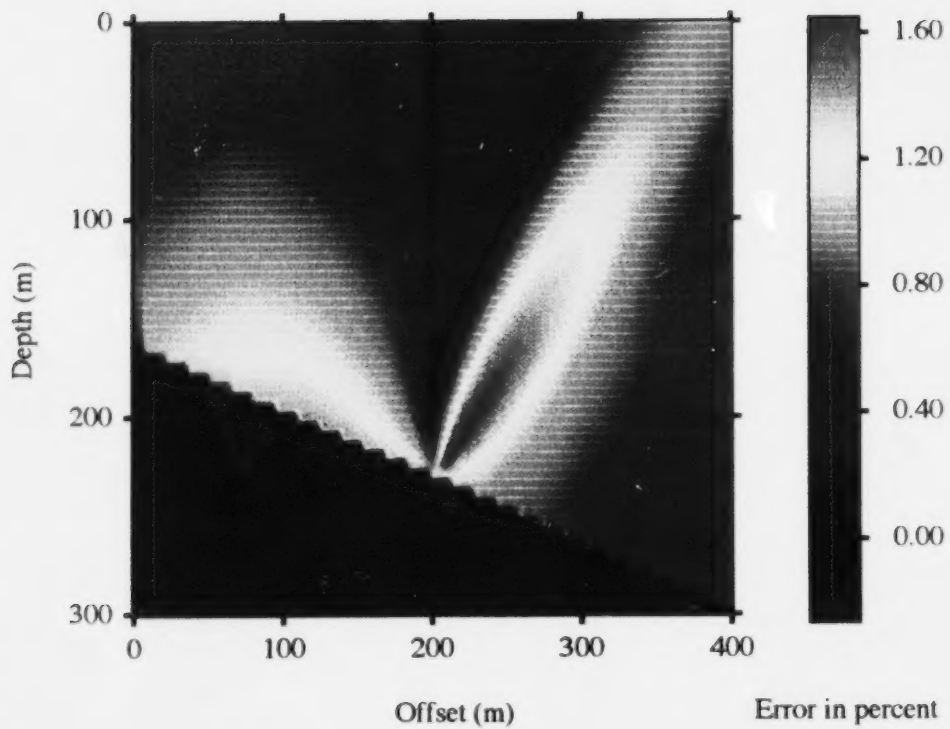


FIG. 3.16 (e). The relative percentage error map of the calculated reflection traveltimes for the dipping structure shown in figure 3.16 (a). Errors are coded by varying levels of colours.

(c). Syncline

The last model shows a syncline structure. Figure 3.17 (a), (b), (c), (d) and (e) illustrate the model structure, the first arrival wavefronts, the wavefronts associated to the incident and reflected waves, the reflections observed at the surface and the error in percent for the calculated reflected traveltimes in the model. This syncline reflecting interface is constructed by three flat segments and two conjugate dipping segments as illustrated in Figure 3.17 (a). The dip angle is 26.5° in the model. As a result, the errors are distributed in a similar manner to those for the dipping case: the maximum errors occur immediately above the dipping reflecting segments.

Figure 3.17 (d) only shows the first arrivals of the waves reflected from the syncline interface. The bowtie-shaped multiple reflection traveltimes which commonly occur in seismic data may also be obtained by calculating the traveltimes of the waves reflected from each of the interface segments, respectively. One example of the multiple reflection traveltimes for the given syncline structure is illustrated in Figure 3.18. In the figure the two dashed curves and the dotted curve represent the traveltimes of the waves reflected from the two conjugate dipping interface segments and the lower horizontal interface segment, respectively. The solid curve represents the first reflected arrival traveltimes, which is identical to that shown in Figure 3.17 (d). Figure 3.18 illustrates that for the structure and the source position the first reflection arrival traveltimes observed at the surface at the offset from 150 m to 250 m are the reflections from the two dipping segments. The remainder of the arrivals are the reflections and diffractions from the upper horizontal segments and the two upper corners of the syncline structure,

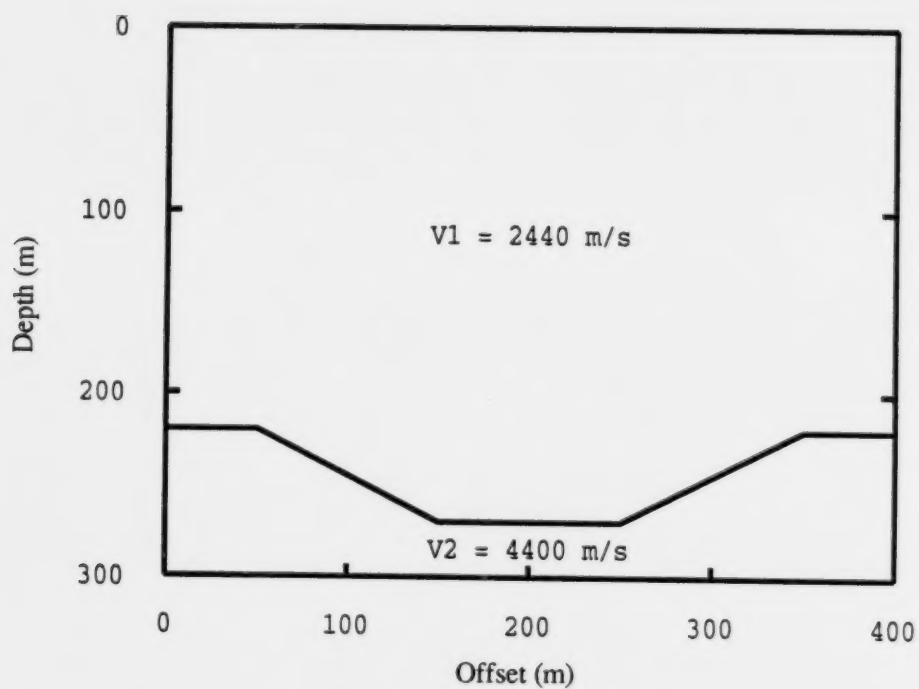


FIG. 3.17 (a). Velocity model for a syncline structure. Three flat and two conjugate dipping segments construct the syncline interface. The source is located at (200,0) of the model.

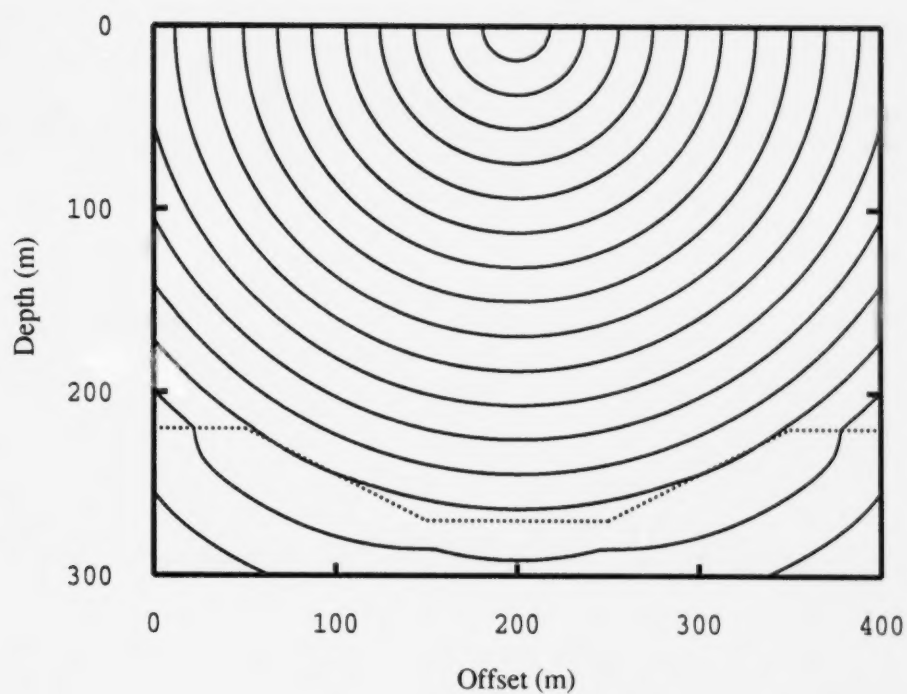


FIG. 3.17 (b). First arrival traveltimes contours for the syncline structure shown in figure 3.17 (a). The time intervals between the contours are 7.7 ms. The dashed line represents the interface.

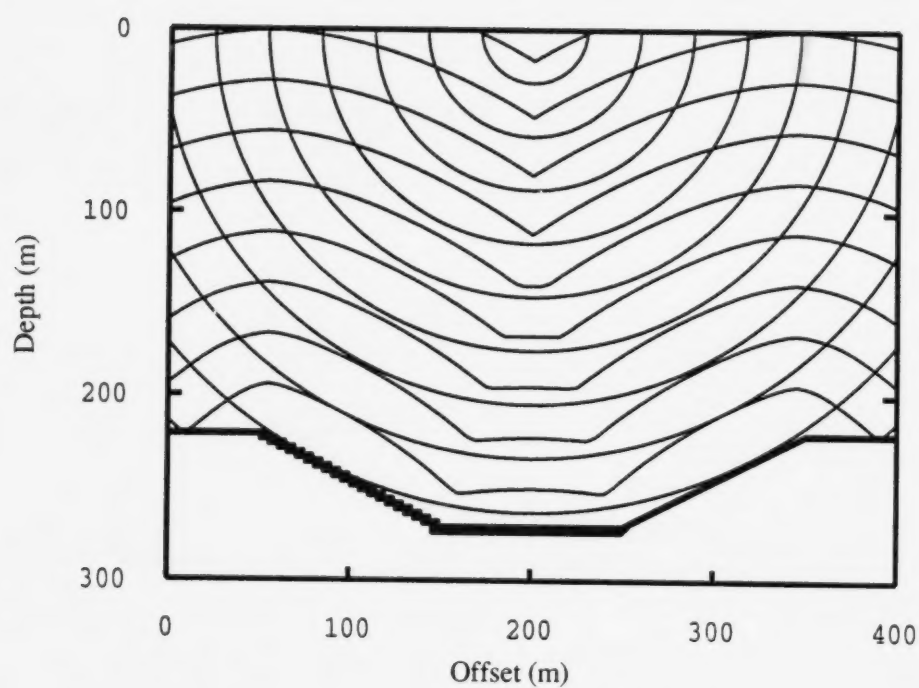


FIG. 3.17 (c). Traveltime contours of the incident and reflected waves for the syncline structure shown in figure 3.17 (a). The time intervals between the contours are 12.0 ms.

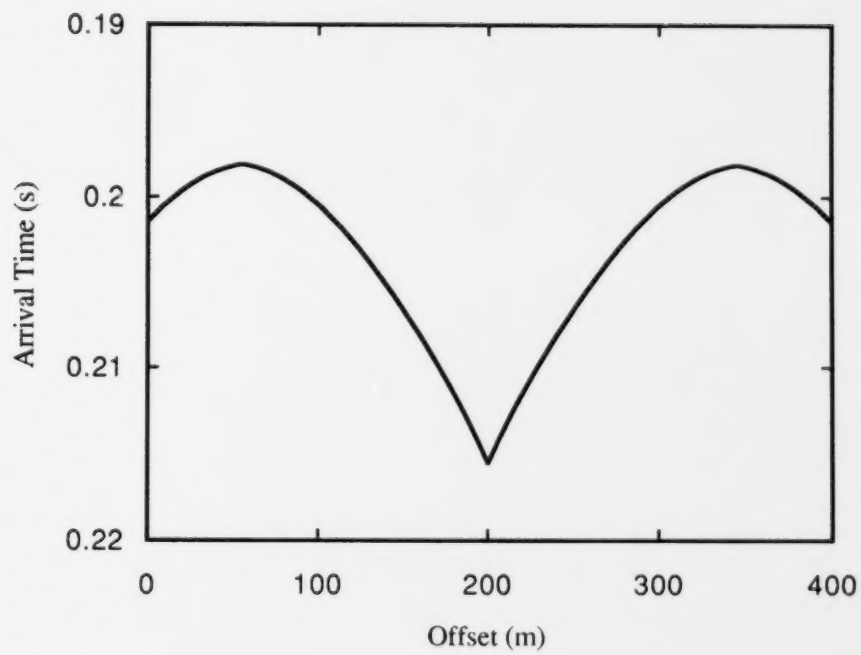


FIG. 3.17 (d). Arrival traveltimes reflected from the syncline interface shown in figure 3.17 (a) and observed at the surface.

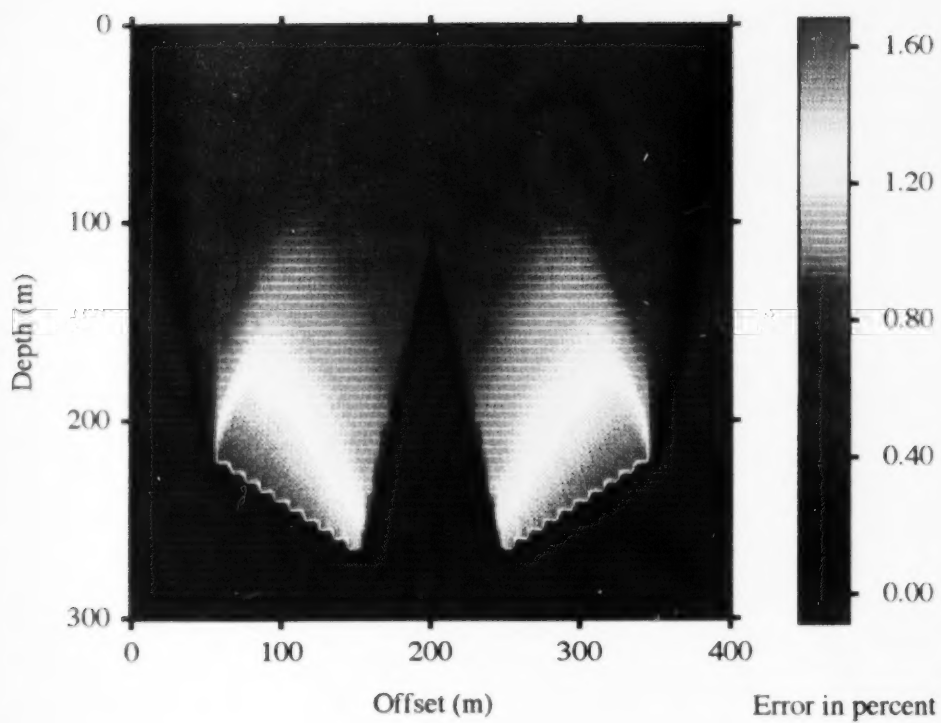


FIG. 3.17 (e). The relative percentage error map of the calculated reflection traveltimes for the syncline structure shown in figure 3.17 (a). Errors are coded by varying levels of colours.

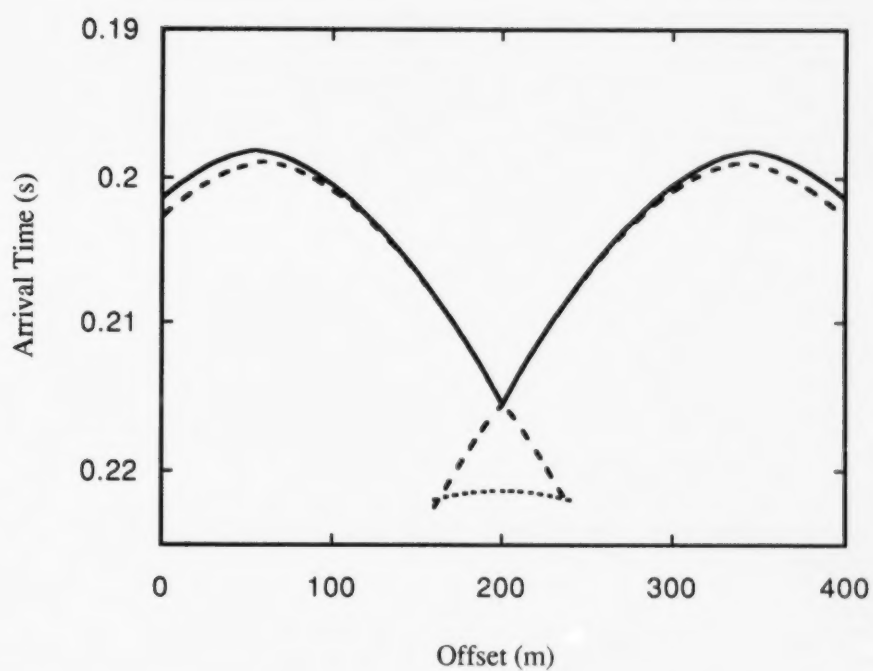


FIG. 3.18. Multiple arrival traveltimes reflected from the syncline interface shown in figure 3.17 (a) and observed at the surface. The two dashed curves and the dotted curve represent the traveltimes of the waves reflected from the two conjugate dipping interface segments and the lower horizontal interface segment, respectively. The solid curve represents the first reflected arrival traveltimes.

respectively. The reflections from the lower horizontal interface segment appear as later arrivals (Figure 3.18).

Note that all the error estimates illustrated above are obtained by the comparison to theoretical results calculated by straight forward application of Snell's law via ray tracing. The maximum errors caused by modeling a dipping interface with steps are estimated as described below: Assuming the dimensions of the rectangular cells have the horizontal and vertical sides of length Δx and Δz , respectively (Figure 3.19), and the slownesses are S_{slow} and S_{fast} on either side of the interface, the maximum traveltime error which occurs along a diagonal through a cell corner is

$$\Delta t_{max} = (\Delta x^2 + \Delta z^2)^{1/2} \cdot (S_{fast} - S_{slow}) \quad (3.12)$$

For a cell in the dimensions of 5 by 2.5 meters and the velocities of 2440 m/s and 4400 m/s, the error is 1 millisecond which permits the accuracy requirement in our current modeling stage.

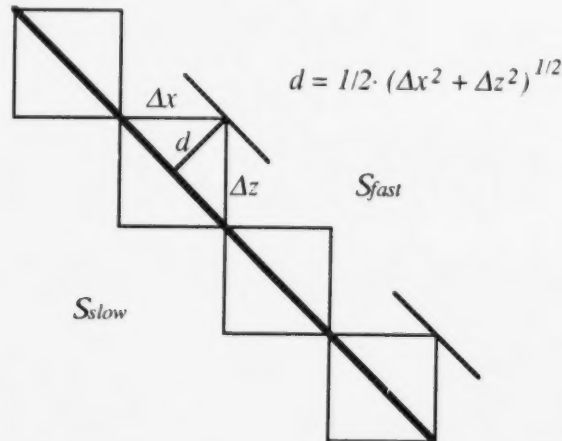


FIG. 3.19. Reflected traveltime error due to modeling an inclined horizon as a series of steps.

Over all, the modified Schneider et al.'s method still retains many of the advantages of the newly developed leading wavefront traveltime calculation technique, i.e., low computing cost and high accuracy even in a complex geological structure. Further, the reflected traveltimes can be calculated with a very little extra cost in the computing time. This method has potential application in other seismic techniques, for example, the geometric amplitude of the waves associated with first arrivals can be obtained with this time information (Vidale et al., 1990), and in the next chapter this method is applied to a synthetic VSP traveltime tomography study.

Chapter 4 A Synthetic VSP Tomographic Inversion Study

4.1 Introduction

In this chapter, the reflection traveltime calculation technique described in Chapter 3 is applied to a simple synthetic vertical seismic profiling (VSP) tomographic inversion study. Some existing algebraic inversion algorithms which produce the resulting tomographic data set are also studied. These are applied to the case of VSP tomography on a given velocity structure similar to that encountered in the shallow oil sands formations near Ft. McMurray. Tomography, developed for x-ray techniques in the medical sciences in 1970s, is a method of using observations of differing wave parameters such as amplitude integrated over a medium to find the two or three dimensional distribution of a physical property within the medium (Aki et al., 1980). It belongs to a broad class of problems in seismology called inverse problems which essentially deal with the question of relating a set of observed seismic waves to the source characteristics and the physical structure through which the seismic waves propagated. Another broad class of problems in seismology are known as forward problems in which the arrival times, the amplitudes and the phases of seismic waves through a structure already defined are calculated. This class can be solved directly provided the basic physical process is understood. For example, the traveltimes can be found for a given velocity structure (Chapters 2 and 3) using the eikonal equation or Fermat principle depending upon the type of calculation which is required.

Tomography can further be classified as either traveltime tomography (based on the eikonal equation) (Aki et al., 1980; Stewart, 1991) or diffraction tomography (based on the wave equation) (Wu et al., 1987; Pratt et al., 1991). Traveltime tomography is a method which uses the observed direct and reflected arrival traveltimes at a series of spatial distributed points to reconstruct 2- or 3-dimensional velocity structure in which seismic waves corresponding to these arrivals propagate (Aki et al., 1980; Stewart, 1991). In diffraction tomography the scattered (or diffraction) field is used to reconstruct the object (Wu et al., 1987; Pratt et al., 1991). Since the full information content of the recorded waveforms is used in diffraction tomography, it is a more accurate method. Traveltime (ray trace) tomography works well under the following two conditions (Wu et al., 1987). First, the velocity field being examined must vary slowly on the scale of the source wavelengths as the method relies on a high frequency assumption of ray theory. In this theory, seismic energy is completely guided by the ray paths and there is no scattering. Further, the phase delay is linear with the frequency, the source wavelet is not distorted, and the seismic events are completely characterized by the traveltimes. Secondly, the source-geophone geometry must provide a wide angular coverage of a medium as the result of no scattering. In other words, a sufficient number of rays must pass through all portions of the geometry in order to accurately reconstruct the geological structure. When either of the requirements breaks down, wave-equation tomography provides a better image. In the present study we are attempting to improve the resolving power of the VSP geometry by incorporating reflected waves into the analysis which give a more uniform coverage.

As noted in equations 2.7 and 2.8, the seismic wave traveltime is the integral of the slowness along a ray path which is guided by the velocity changes for the geological structure. Therefore, to convert to the velocity structure from the observed seismic traveltimes is a nonlinear problem. In practice, however, a reference velocity is first assumed; this is often based on previous knowledge of the geology obtained from well logs or a previous seismic survey. This reference velocity is used to guide the ray paths initially. The traveltime differences, or time delays, between the actual observed arrivals and the traveltimes obtained from the ray tracing through the reference velocity structure are used to calculate the slowness perturbations. This produces a proximate but linear procedure suitable for the application of tomography.

Often in traveltime tomography, it is adequate to use a straight ray assumption as an approximate technique when the changes of velocities throughout the geology are relatively small (McMechan, 1983). However, it is very difficult or impossible to predict when this assumption is no longer valid. Therefore, there is a compelling reason to use curved ray tomography in all situations since they are based on a more realistic model of wave propagation through a medium of variable velocities. In curved ray tomography, the velocity field is continually updated requiring that the rays are retraced between all source-receiver pairs on each cycle of the imaging procedure (George et al., 1987; Bregman et al., 1989a). This ray-tracing constitutes a large part of the computational cost of curved ray tomography. One problem is that conventional ray tracing methods such as the shooting or bending methods have difficulty in handling head waves and diffracted waves and some special techniques must be applied in these cases (Doornbos, 1988).

As a result, the conventional ray tracing techniques are computationally inefficient for tomography when the structure may need to be updated several hundred times before an acceptable answer is found. In this study, we circumvent the problems associated with conventional ray tracing by calculating traveltimes to all points of a two dimensional slowness field with Schneider et al.'s (1992) method as modified in Chapter 3. Ray paths are then generated by following the gradient (i.e., the steepest descent) direction through the computed traveltimes from each receiver back to the source (Vidale, 1988). This method yields the ray paths of all first arrivals regardless of whether they were direct, critically refracted, or diffracted as well as the reflected arrivals. Moreover, an arbitrary recording geometry is easily accommodated since arrival times are calculated throughout the velocity field. A last, but not least important, point is that the forward modeling method uses a discretized grid of constant velocity cells which can be the same as the grid used in the tomographic inversion.

In addition to the computational time and accuracy requirement for forward modeling, tomography also requires that a system of linear algebraic equations are solved in order to obtain an improved velocity structure. This solution may exhibit erratic and unphysical behavior due to noise in the traveltime data and ill-conditioning of the equations. Therefore, some form of regularization is often used to stabilize the inversion procedure (Bishop et al., 1985; Bregman et al., 1989a; Macrides et al., 1988). Scales (1987) and Phillips et al. (1991) provide good reviews of some popular algebraic inversion methods.

In the following sections, the theoretical bases for traveltime tomography are briefly reviewed. Then a comparison of three existing

methods: singular value decomposition (SVD); algebraic reconstruction technique (ART); and simultaneous iterative reconstruction technique (SIRT) is first studied on a horizontal layered VSP case using a straight ray assumption. This is carried out primarily to test the stability of the differing solutions for the relatively poorly posed problem of the VSP tomography in which the range of illumination angles is severely restricted. Finally, an attempt to use the newly developed ray tracing method by applying the reflection traveltime calculation technique noted in Chapter 3 on a simple synthetic VSP curved ray tomography is implemented.

4.2 Theoretical Basis

As noted earlier, traveltime tomography based on the ray theory is a nonlinear problem since the ray paths used to reconstruct a geological structure rely on this geological structure. However it may be linearized by introducing a reference geological structure from which perturbations are determined. Tomography is then carried out via the solution to a linearized system of equations. The theoretical basis for this procedure is introduced in this section.

4.2.1 Linearization of the Eikonal Equation

It is well known that under the high frequency assumption, the traveltime for a seismic wave propagating through a slowness field $s(\mathbf{x})$ satisfies the eikonal equation

$$(\nabla t(\mathbf{x}))^2 = s^2(\mathbf{x}) \quad (4.1a)$$

which may be explicitly written in a Cartesian coordinate (x, y, z) as

$$\left(\frac{\partial t(\mathbf{x})}{\partial x}\right)^2 + \left(\frac{\partial t(\mathbf{x})}{\partial y}\right)^2 + \left(\frac{\partial t(\mathbf{x})}{\partial z}\right)^2 = s^2(\mathbf{x}) \quad (4.1b)$$

where $t(\mathbf{x})$ is the travelttime of the associated seismic wave to a given point $\mathbf{x}$ in the medium. This nonlinear partial differential equation can be linearized using a small slowness perturbation $\Delta s(\mathbf{x})$ to the reference slowness $s_0(\mathbf{x})$. Assume that the slowness is given by

$$s(\mathbf{x}) = s_0(\mathbf{x}) + \Delta s(\mathbf{x}) \quad (4.2)$$

Then the travelttime function may also be split into two constituent parts

$$t(\mathbf{x}) = t_0(\mathbf{x}) + \Delta t(\mathbf{x}) \quad (4.3)$$

with the reference model obeying

$$(\nabla t_0(\mathbf{x}))^2 = s_0^2(\mathbf{x}) \quad (4.4)$$

Substituting equations (4.2) and (4.3) into equation (4.1) yield

$$\nabla(\Delta t) \cdot \left\{ \left[\frac{\nabla t_0}{s_0} \right] + \frac{1}{2} \left[\frac{\nabla(\Delta t)}{s_0} \right] \right\} = \Delta s \left\{ 1 + \frac{1}{2} \left[\frac{\Delta s}{s_0} \right] \right\} \quad (4.5)$$

Assuming the following two approximations

$$\left| \frac{\nabla(\Delta t(\mathbf{x}))}{s_0(\mathbf{x})} \right| \ll 1, \quad \text{and} \quad \left| \frac{\Delta s(\mathbf{x})}{s_0(\mathbf{x})} \right| \ll 1, \quad (4.6)$$

which indicate that the slowness perturbations are much smaller than the reference slowness and the associated time perturbations change very slowly corresponding to the change of the time derived from the reference, equation (4.5) is reduced to the linearized eikonal equation

$$\nabla(\Delta t(\mathbf{x})) \cdot \mathbf{l}_0(\mathbf{x}) = \Delta s(\mathbf{x}) \quad (4.7)$$

where $\mathbf{l}_0(\mathbf{x}) = \nabla t_0(\mathbf{x}) / s_0(\mathbf{x})$ is a dimensionless unit vector indicating the ray direction at point $\mathbf{x}$ in the medium. Note that the left hand side of equation (4.7) is the directional derivative of $\Delta t(\mathbf{x})$ in the ray path direction $\mathbf{l}_0(\mathbf{x})$. Hence, this expression may be rewritten as

$$\frac{d(\Delta t(\mathbf{x}))}{dl_0} = \Delta s(\mathbf{x}) \quad (4.8)$$

where dl_0 is an infinitesimal displacement along the ray path. Finally, integrating equation (4.8) yields the familiar result for the traveltime perturbation

$$\Delta t(\mathbf{x}) = \int_{L_0(\mathbf{x})} \Delta s(\mathbf{x}) dl_0 \quad (4.9)$$

where $L_0(\mathbf{x})$ is the ray path through the reference medium. Integrating the eikonal equation (4.4) for the reference model yields (equation (2.8))

$$t_0(\mathbf{x}) = \int_{L_0(\mathbf{x})} s_0(\mathbf{x}) dl_0 \quad (4.10)$$

Adding equations (4.9) and (4.10) together yields an alternative linear traveltime equation

$$t(\mathbf{x}) = \int_{L_0(\mathbf{x})} s(\mathbf{x}) dl_0 \quad (4.11)$$

Equation (4.11) relates the traveltime to the medium slowness via the ray tracing through the reference slowness. Note that the perturbation equations (4.2) and (4.3) are used in deriving the above equation.

Equations (4.9) or (4.11) set up the basis for the application of traveltime tomography. However, the discrete forms of these equation are often used in

numerical calculations. The spatial distribution of the slowness and its perturbation can be described by dividing the medium into a number of cells (Figure 4.1). The slowness is constant within each cell and, consequently, rays follow a straight line within each cell. The integrals (4.11) and (4.9) which give the traveltime and traveltime perturbation along the i th ray path (Figure 4.1), respectively, are rewritten in segmented fashion as

$$t_i = d_{i1}s_1 + d_{i2}s_2 + d_{i3}s_3 + \dots + d_{in}s_n \quad (4.12)$$

$$\Delta t_i = d_{i1}\Delta s_1 + d_{i2}\Delta s_2 + d_{i3}\Delta s_3 + \dots + d_{in}\Delta s_n \quad (4.13)$$

where s_j and Δs_j ($j=1, 2, 3, \dots, n$) are the slowness and its perturbation for the j th cell as illustrated in Figure 4.1, respectively. d_{ij} ($j=1, 2, 3, \dots, n$; $i=1, 2, 3, \dots, m$) is the length of the line segment of the i th ray in the j th cell, and n and m are the number of cells and traces (arrivals), respectively. Note that many of these d_{ij} have zero value as each ray will only propagate through a limited number of cells. t_i and Δt_i are the observed arrival traveltime and the associated time delay for i th ray, respectively. For a collection of ray paths between m source-receiver pairs (i.e., m arrivals), the equations (4.12) and (4.13) are expressed as

$$\sum_{i=1}^m t_i = \sum_{i=1}^m \left\{ \sum_{j=1}^n [d_{ij} \cdot s_j] \right\} \quad (4.14)$$

and

$$\sum_{i=1}^m \Delta t_i = \sum_{i=1}^m \left\{ \sum_{j=1}^n [d_{ij} \cdot \Delta s_j] \right\} \quad (4.15)$$

or in matrix forms

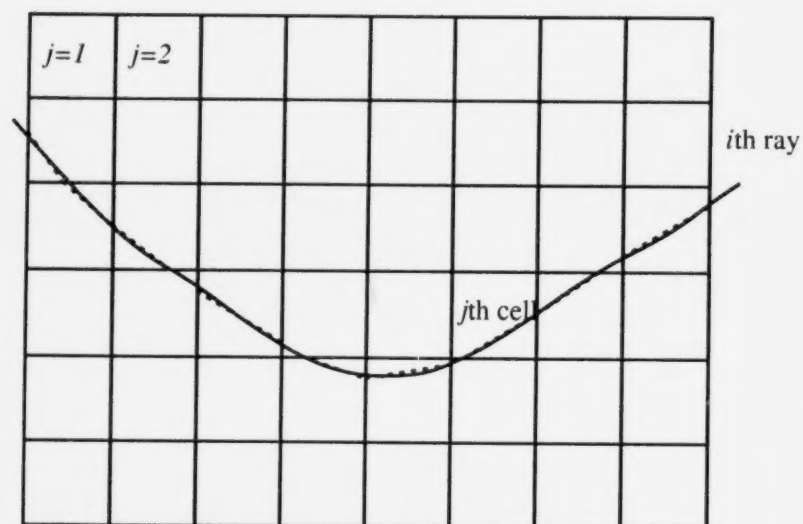


FIG. 4.1. The discrete integral of *i*th ray path. The solid line shows the ray path whereas the segment of the dashed straight line in each cell is the discretized ray segment in that cell.

$$\mathbf{t} = \mathbf{D}(\mathbf{s}_0) \cdot \mathbf{s} \quad (4.16)$$

and

$$\Delta \mathbf{t} = \mathbf{D}(\mathbf{s}_0) \cdot \Delta \mathbf{s} \quad (4.17)$$

where $\mathbf{t}$ and $\Delta \mathbf{t}$ are the traveltimes and the time delay vectors with the elements of t_i and Δt_i , respectively. $\mathbf{s}$ and $\Delta \mathbf{s}$ are the slowness and the its perturbation vectors with the elements of s_j and Δs_j , respectively. $\mathbf{s}_0$ is the reference slowness vector and $\mathbf{D}$ is the length matrix of the ray segment with the elements of d_{ij} .

4.2.2 Solving a Least Square Problem

In tomographic inversion, a model perturbation $\Delta \mathbf{s}$ is sought such that the improved model $\mathbf{s} = \mathbf{s}_0 + \Delta \mathbf{s}$ approximates the true slowness model associated with the observed traveltimes. The traveltime difference vector $\Delta \mathbf{t}$ is given by $\Delta \mathbf{t} = \mathbf{t}_{obs} - \mathbf{t}_0$, where $\mathbf{t}_{obs}$ is the observed traveltimes vector and $\mathbf{t}_0$ is a vector of traveltimes calculated from the reference slowness model $\mathbf{s}_0$. In principle, equation (4.17) can be solved for the required model perturbation. In practice, however, difficulties in the solution arise because of the following problems existing in the acquired data:

- (a) the large number of equations; the total number of equations is the number of rays multiplied by the number of cells;
- (b) the sparse matrix; i.e., relatively few elements of the ray matrix $\mathbf{D}$ are not equal to zero since a ray passes through only a few cells;
- (c) the over determined problem; in most of the cases there are more rays (data) than cells (unknowns);
- (d) the non-unique solutions; the rank($\mathbf{D}$) is less than the number of cells,

i.e., less number of data than that of unknown are independent;

(e) the inconsistent system of equations; there always exist various noise in the data (that is, errors in the system of the equations) and the physical model deviates from the true geology because of the discretization of the cells.

Overall, there are no exact analytic and/or numerical solutions to the linear system of equations (4.17) and a minimum misfit solution is usually sought.

Mathematically, these type of problems in finding a minimum misfit or a minimum norm for a vector are classified as least square problems and are usually expressed as

$$|t_{obs} - D(s) \cdot s| = \text{minimum} \quad (4.18)$$

where $||$ is the norm of a vector. That is, a solution of 4.18 will be accepted provided it gives the minimum amount of error. Numerous techniques dealing with the least square problem have been developed in seismology and one can refer to Lanczos (1961) or Aki and Richards (1980) for full discussions.

4.3 A Comparison of ART, SIRT and SVD Methods

Many conventional least square techniques are not effective when applied to tomography. The reason is that even though the original ray matrix D is very sparse (i.e., contains many zeros), it will become a dense one during the standard solution process since the deconvolved matrixes are filled up with non-zero elements. Consequently, a large memory capacity is required on a computer. This requirement makes these standard inversion techniques impractical for numerical calculations. Therefore alternative techniques are required so that mathematical procedures can be carried out

for tomography with a minimum computer capacity demand. Phillips et al. (1991) and Scales (1987) provide good reviews of numerical inversion methods.

In this section, three different, and well known, tomographic inversion techniques: the algebraic reconstruction technique - ART (Golden, 1974), the simultaneous iterative reconstruction technique - SIRT (Ivasson, 1986), and singular value decomposition - SVD (Stork, 1992a, b) are applied. Since the initial motivation is to study the stability of the different numerical techniques themselves, a hypothetical VSP tomography is carried out via a straight ray path assumption on a horizontal layered medium.

4.3.1 A Review of ART, SIRT, and SVD methods

The algebraic reconstruction technique (ART) is based on Kaczmarz (1937) method of projections. From the geometrical point of view, the image represented by the slowness perturbation vector $\Delta s = (\Delta s_1, \Delta s_2, \dots, \Delta s_n)$ may be considered to be one single point in a n -dimensional space where each of the equations (4.15) represents a hyperplane (Figure 4.2). When a unique solution to these equations exists, the intersection of all the hyperplanes is a single point in this n -dimensional space and gives the solution. However, as there is no exact solution to the linear system of equations (4.15) or (4.17), a minimum misfit solution region is then sought instead (Figure 4.3), that is, the best solution in the region where the hyperplanes come close together. ART is an iterative procedure to find this intersection or region using Kaczmarz method of projections.

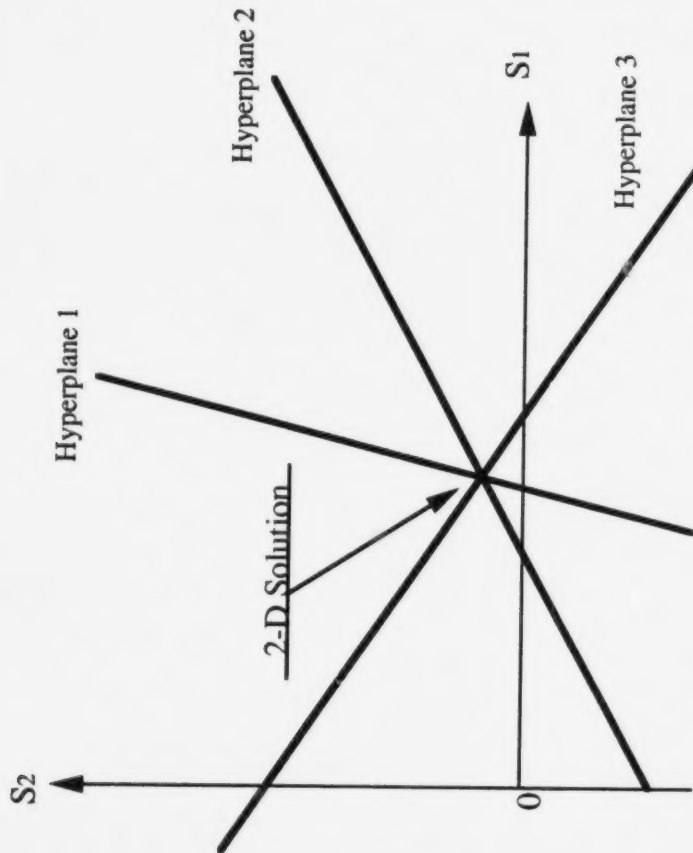


FIG. 4.2. Geometric solution to linear system of equation (4.15) or (4.17). The intersection of all the hyperplanes gives the solution.

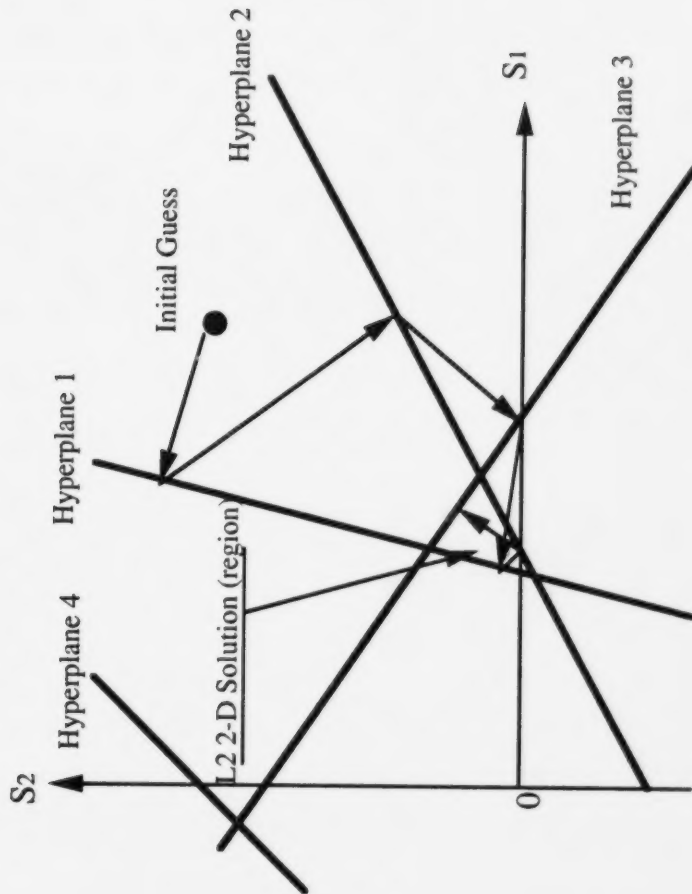


FIG. 4.3. The L2 geometric solution to linear system (4.15) or (4.17) using ART method. A minimum L2 misfit solution region (i.e., the triangle region) is sought. (After Kanasewich, 1992)

The principle of the ART method can be described geometrically as shown in Figure 4.3 (Macrides et al., 1988; Kanasewich, 1992). An arbitrary initial guess of the slowness perturbation, $\Delta s^{[0]} = (\Delta s_1^{[0]}, \Delta s_2^{[0]}, \dots, \Delta s_n^{[0]})$, is projected on the hyperplane represented by the first of equations (4.15), giving the first projection $\Delta s^{[1]}$ to be

$$\Delta s^{[1]} = \Delta s^{[0]} - \frac{(\mathbf{d}_1 \cdot \Delta s^{[0]} - \Delta t_1) \mathbf{d}_1}{\mathbf{d}_1 \cdot \mathbf{d}_1}, \quad (4.19)$$

where $\mathbf{d}_1 = (d_{11}, d_{12}, \dots, d_{1n})$. The obtained $\Delta s^{[1]}$ is then projected on to the hyperplane represented by the second of the equations (4.15) and the second projection $\Delta s^{[2]}$ is obtained. This process is repeated with the third hyperplane, and so on. Consequently, the projection on the j th hyperplane, i.e., the equation for the j th ray and associated time delay, may be obtained from that on the $(j-1)$ th hyperplane by

$$\Delta s^{[j]} = \Delta s^{[j-1]} - \frac{(\mathbf{d}_j \cdot \Delta s^{[j-1]} - \Delta t_j) \mathbf{d}_j}{\mathbf{d}_j \cdot \mathbf{d}_j}, \quad (4.20)$$

where $\mathbf{d}_j = (d_{j1}, d_{j2}, \dots, d_{jn})$. The process of taking projections on different hyperplanes is continued until $\Delta s^{[m]}$ is obtained by taking the projection on to the hyperplane represented by the last of the equations (4.15). The above process is called a cycle. The second cycle starts by projecting $\Delta s^{[m]}$, the last projection in the first cycle, on to the first hyperplane again. This process continues until all the m hyperplanes have been cycled through. This iteration process is geometrically illustrated in Figure 4.3. The number of iterations depends the initial guess, the complexity of a model, and the accuracy requirement. It may be shown (Macrides et al., 1988; Tanabe, 1971) that

$$\lim(\Delta s^{(km)}) = \Delta s_s, \quad (k \rightarrow \infty) \quad (4.21)$$

if a unique solution Δs_s to the system of equations (4.15) exists, where K is the iterative cycle number. In practice, however, a least square solution to equations (4.15) is obtained as noted earlier. Golden (1974) also gave an early detailed discussion on the algebraic reconstruction technique.

In general, ART is stable and converges quickly. However, sometimes erroneous data exist (e.g., the 4th hyperplane in Figure 4.3) and these erroneous data always tend to pull the projection away from the true solution. A conventional technique for handling this problem is to simply ignore the erroneous data if possible. Ivansson (1986) provided an advanced variation of ART known as the simultaneous iterative reconstruction technique (SIRT). SIRT improves the convergence by projecting the initial guess to all the hyperplanes. The obtained projections are then averaged and the averaged result is used as a correction to the initial guess. This correction (i.e., a new point in the n -dimensional space) is again projected to the hyperplanes simultaneously and the newly obtained projections are averaged. This iterative process is repeated until the solution to equations (4.15) is obtained. However, as a trade-off of the stability, SIRT loses the speed of convergence to a solution as compared to ART.

From the technical point of view, both ART and SIRT are computational efficient since the system is solved using only one row of the D matrix at a time and no large computer central memory for the matrix is required. It should be noted that the conjugate gradient algorithm also has the similar attractive characteristics (Nolet, 1987; Scales, 1987). Finally, SIRT can also be applied to a parallel computer to overcome the speed disadvantages.

As noted earlier, tomography is a singular or ill-conditioned problem, i.e., some eigenvalues of the ray matrix D are so small that their values are dominated by the observational and computer roundoff errors. A powerful mathematical tool to solve these types of problems is singular value decomposition - SVD (Press et al., 1988; Stork, 1992a, b). The SVD method is based on a series linear algebra theorems whose proof are beyond the present scope, its principle is to decompose the singular or very small eigenvalues in the ray matrix D to make the solution process more stable. A main difficulty of SVD is that even if the original ray matrix is very sparse, the deconvolved matrixes are filled up with non-zero elements. Therefore, large computer memory capacity and more powerful computers are required because the size of the system of equations for tomographic inversion problems are usually very large (Scales, 1987; Bregman et al., 1989a; Stork, 1992b). Stork (1992b) made a good effort to develop a more practical SVD algorithm by introducing symmetry into his model so that the model size that can be computed with SVD is significantly increased. However, only the standard SVD method (Press et al, 1988) is applied in our study.

4.3.2 A Hypothetical VSP Tomography Study on the Three Methods

Using Straight Ray Paths

A hypothetical VSP tomographic inversion is carried out for a horizontal layered velocity model by applying the above three different numerical methods. The velocity is constant within each layer as illustrated in Figure 4.4. Since the initial motivation is to test the stability and converging speed of the different inversion techniques themselves, the forward modeled traveltimes are calculated using a simplifying assumption of a straight ray path between source and receiver. This straight ray tracing

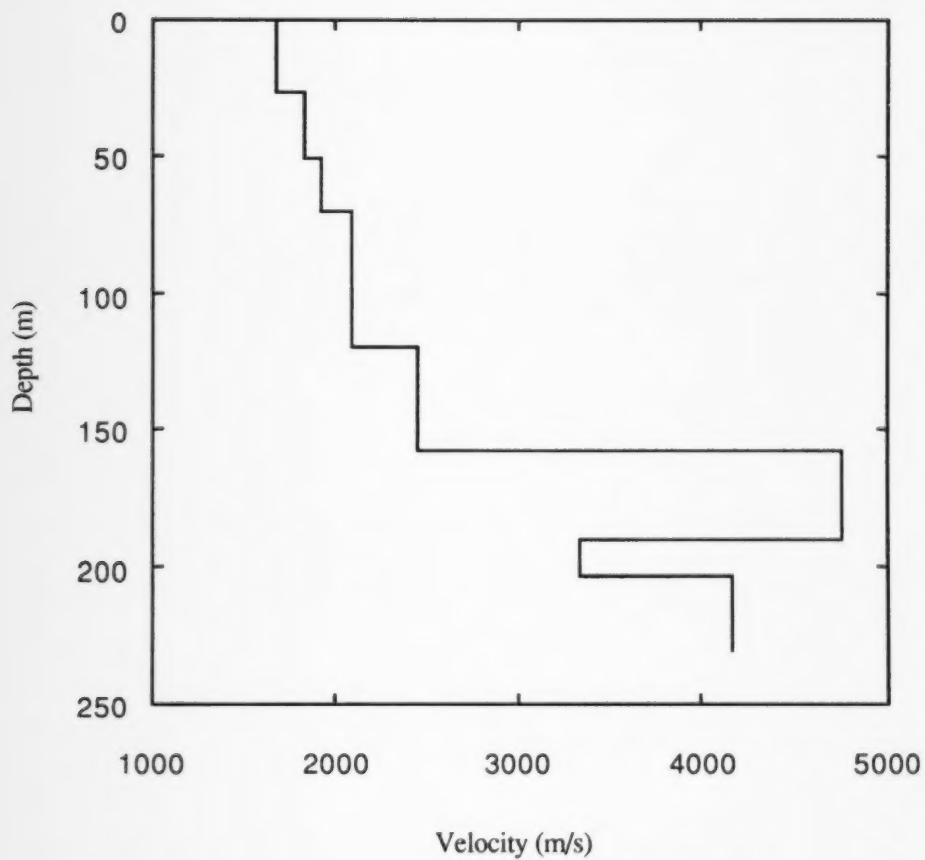


FIG. 4.4. A horizontal layered velocity structure. The velocity is constant in each layer.

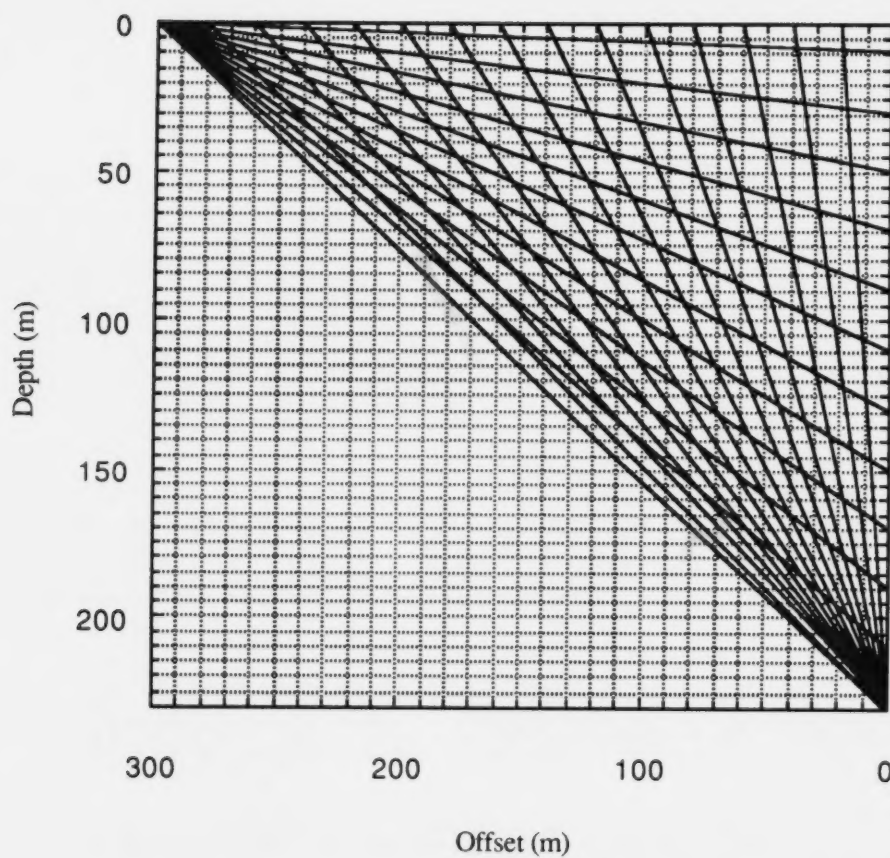


FIG. 4.5. A straight raypath assumption. This assumption is used in both the forward modeling and the inverse imaging process.

assumption is also applied to the final inversion (Figure 4.5). In the forward modeling, thirty surface sources are set at a 10 meters spacing along a radial line extending from the borehole axis and twenty-three borehole receivers are also at a 10 meters spacing as illustrated in Figure 4.5. This, with 10 m x 10 m image pixels (cells), gives a inversion problem which has 690 equations (arrivals) and 371 unknowns (pixels or cells). Since a straight ray path assumption is used in the model, the region of ray coverage is triangular (Figure 4.5).

The three inversion techniques (i.e., SVD, ART, and SIRT) are applied to solve this over determined system of equations and the resulting velocity images are shown in Figures 4.6 (a), (b), and (c) corresponding to the SVD, ART, and SIRT methods, respectively. The computational CPU times on a SPARC-2 SUN Station and the model misfits E , which represent the convergence and the quality of the reconstruction, for the three techniques are shown in Table 4.1. The misfit E is written as (Kanasewich et al., 1989)

$$E = \sqrt{\frac{1}{m} \sum_{i=1}^m (t_i^{obs} - t_i^{cal})^2}, \quad (4.22)$$

where t_i^{obs} and t_i^{cal} are observed and calculated arrival times through the image velocity model, respectively, for the i th ray or observed arrival. m is the number of traces. In general, a smaller E value represents a better match between the model and the reconstruction. As shown in Figures 4.6, the three inversion routines all converged and are computational stable. The SVD inversion reconstructs the layered velocity structure more faithfully than the other two techniques (velocities are coded by varying levels of colors). The layers are readily seen in Figure 4.6 (a) whereas substantial smearing results in Figures 4.6 (b) and (c), especially in Figure 4.6 (c). This

conclusion is also supported by the misfits E in Table 4.1. Compared with the other two methods, the SVD method is also computational time efficient as shown in Table 4.1. It should be noted, however, that the scale of our inversion model is very small (690 equations with 371 unknowns) so that the SVD method can be implemented without the difficulty of computer capacity. In practice, the requirement for a large computer memory often limits the use of the SVD method as noted earlier. It also must be pointed out that these results are obtained with "noise-free" data, that is the inputs contain no traveltimes picking errors. 1000 iterations are used for both ART and SIRT techniques in the above inversion calculations. An intensive study on the speed of convergence to a solution for the SIRT method is also carried out and the resulting model misfits E are shown in Table 4.2 for a set of different iterations.

The above results are obtained with "noise-free" data which are unrealistic in practice. Therefore, studying the noise behaviors of the three inversion techniques is very instructive. Figures 4.7 (a) and (b) illustrate the resulting velocity images corresponding to 0.1 ms random noises (i.e., an up to 1.2% relative error) in the initial data for the SVD and ART methods, respectively. The associated misfits are shown in Table 4.1. Figures 4.7 indicate that the ART method is more stable for this model under the given noise level whereas the SVD method can not faithfully reconstruct the image, especially in the region where poor angular ray coverage takes place (e.g., near the diagonal line of the model). As noted earlier, a smaller E value generally corresponds to a better match between the model and the reconstruction. In this example, however, the E value is smaller for the SVD method than that for ART (Table 4.1), though ART gives a better image (Figures 4.7 (a) and (b)).

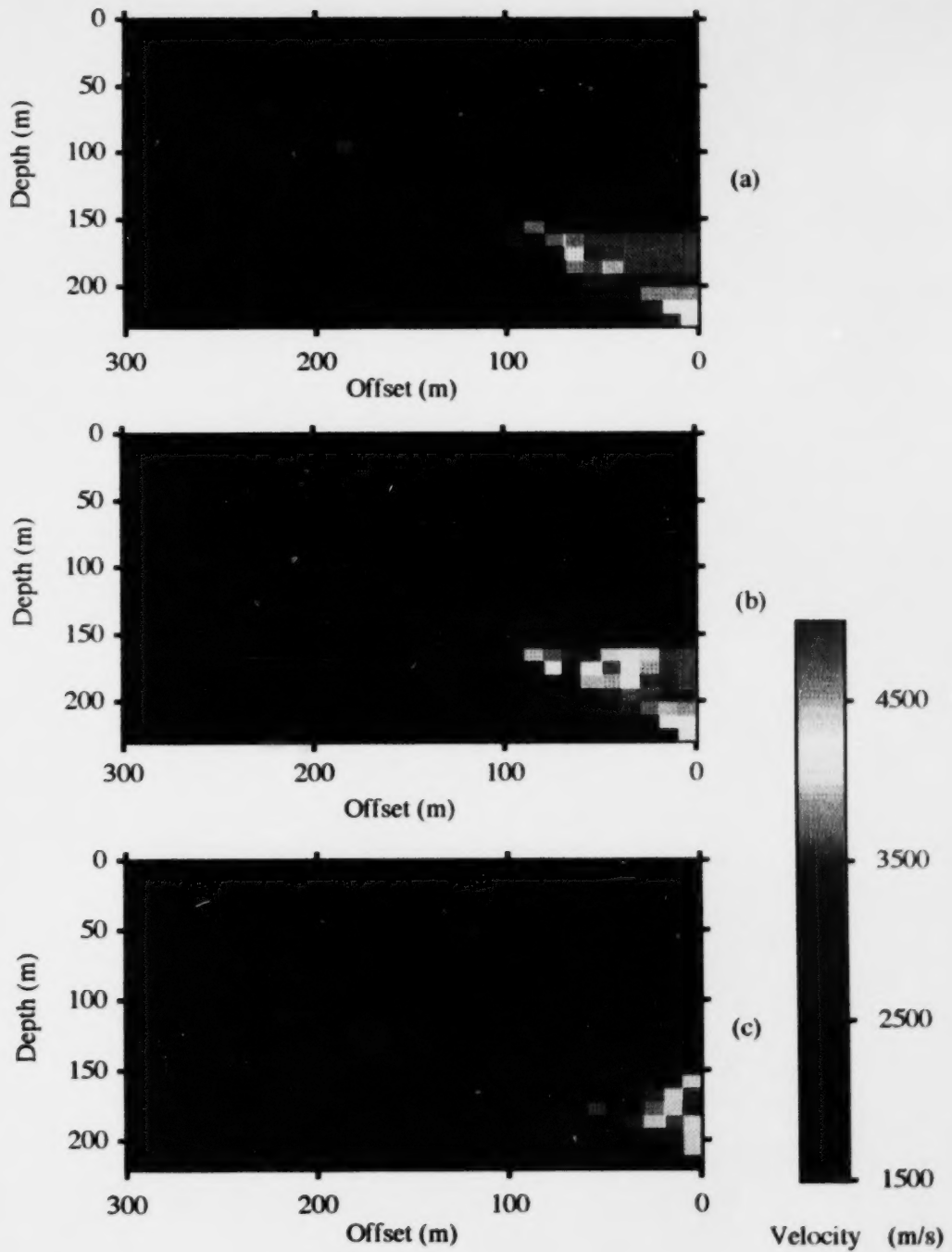


FIG. 4.6. Velocity images (noise-free) of the structure shown in figure 4.4 using a straight raypath assumption and (a) SVD method, (b) ART method, and (c) SIRT method. Different scales are used in horizontal and vertical directions.

Table 4.1. The comparison of the efficiency, convergence, and the stability of the three methods.

	SVD	ART	SIRT
CPU time (min.)	14.5	21.6	31.7
misfit E (ms) (noise-free)	3.3×10^{-4}	1.1×10^{-2}	0.99
misfit E (ms) (noise level: 0.1 ms)	3.9×10^{-2}	4.9×10^{-2}	0.99

Table 4.2. The convergence study of the SIRT method.

# iteration	1000	2000	4000	8000
misfit E (ms) (noise-free)	0.99	0.58	0.34	0.22

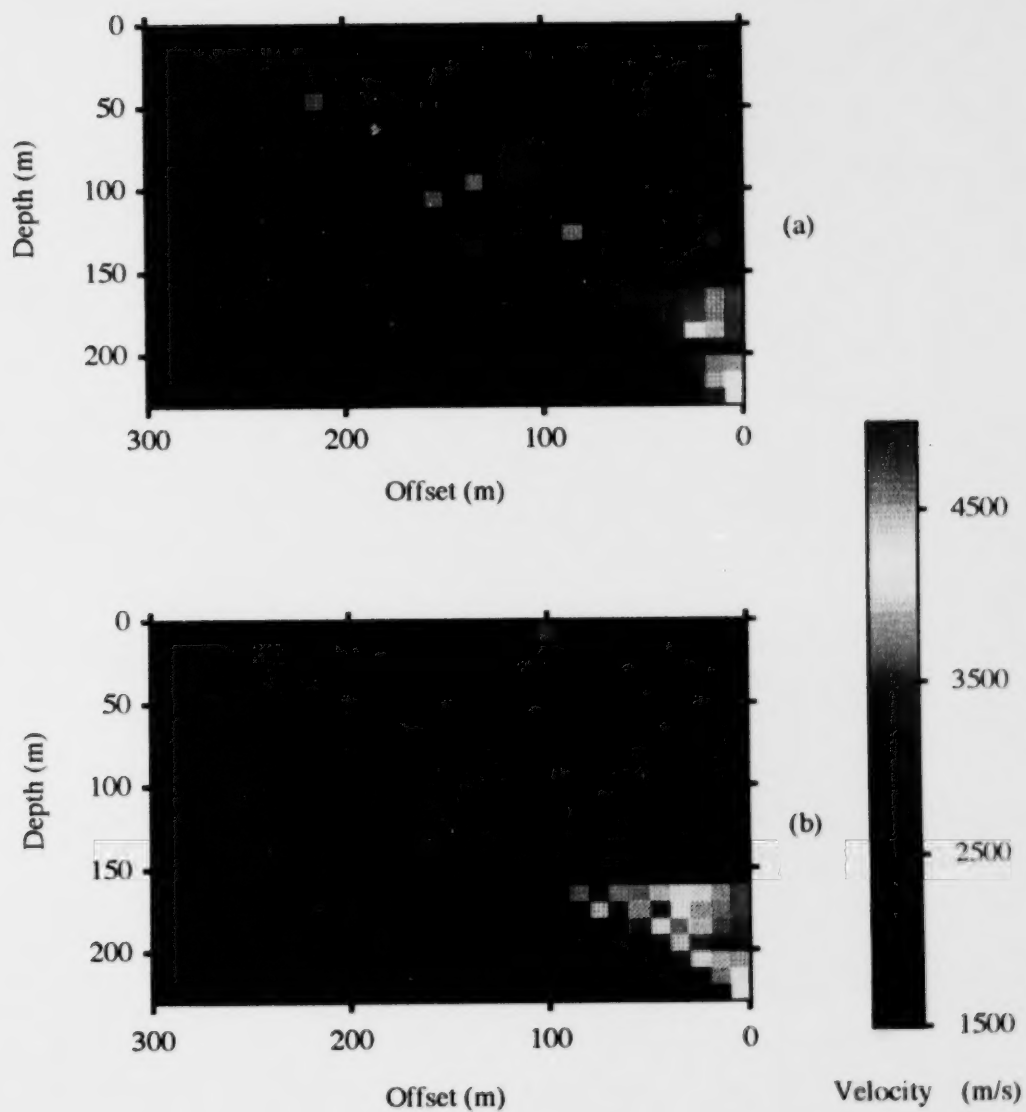


FIG. 4.7. Velocity images of the structure shown in figure 4.4 using a straight raypath assumption and (a) SVD method, (b) ART method. 0.1 ms random noises are added to the arrivals.

The deviation from the general theorem raises an important issue in traveltime tomographic inversion, that is the effects of poor angular coverage of rays, called ghosts. Bregman et al. (1989b) present a good discussion on this issue. This ghosting effect can be understood from the fact that a large localized perturbation and a small but more widely distributed one could give the same traveltime observation results since the time delay is the integral of slowness perturbations along a ray path (equation (4.9)). As a result, it is difficult to image (i.e., position) structures oriented perpendicular to the ray paths. The straight ray path assumption used in this model has the result that the rays corresponding to the far-offset source and the deep-depth receiver pairs are nearly parallel to the diagonal line of the model and, unfortunately, they are the only angles at which rays pass through the pixels located near the image edge. Consequently, both the widely distributed slowness perturbations and the few but larger ones (e.g., the pixels represented by high levels of the color codes in Figure 4.7 (a)) give the same time delay. The SVD method fails to reconstruct the image in this case. Table 4.1 also indicates that the SIRT method is not sensitive to this noise level.

Based on the performance, one can conclude that ART method probably is the best one among these three techniques. However, the SVD and SIRT methods are also applicable provided more powerful computers (e.g., a large computer memory for SVD and parallel processors for SIRT) are available.

4.4 Application of the Reflection Traveltime Calculation Method on a Simple Synthetic Cured Ray VSP Tomography Study

It is well known that curved ray paths are based on a more accurate model of the wave propagation through variable velocity media and should

be applied in all situations in traveltime tomography (George et al., 1987; Bregman et al., 1989a). It also has been noted that the angular distribution of ray coverage takes an significant role in the quality of a reconstructed image (McMechan, 1983; Nolet, 1985; Bregman et al., 1989b), or even results in the failure in some regions (Figure 4.7 (a)). In this section, preliminary attempts have been made in solving these two major difficulties related to traveltime tomography using a newly developed technique based on Schneider et al.'s (1992) method to improve the efficiency and accuracy of the ray tracing and using reflection rays to increase the information used for tomographic inversion.

4.4.1 A Newly Developed Ray Tracing Technique

As noted, curved ray paths should be applied in all situations in traveltime tomography since they are based on a more accurate model of the wave propagation. However, this ray tracing constitutes a large part of the computational cost. Conventional ray tracing methods such as the shooting or bending methods are computationally inefficient for curved ray tomography. An efficient alternative route might be to calculate traveltimes to all points of a two (or three) dimensional slowness field and then to generate the ray paths by following the gradient (i.e., the steepest descent) direction (Figure 4.8) through the computed traveltimes from each receiver back to the source (Vidale, 1988, 1990). For example, in this section the new ray tracing technique is applied to two geological models consisting of a horizontal layered model and a similar model which includes an embedded low velocity block. Schneider et al.'s (1992) and the modified reflection methods are used for traveltime calculations.

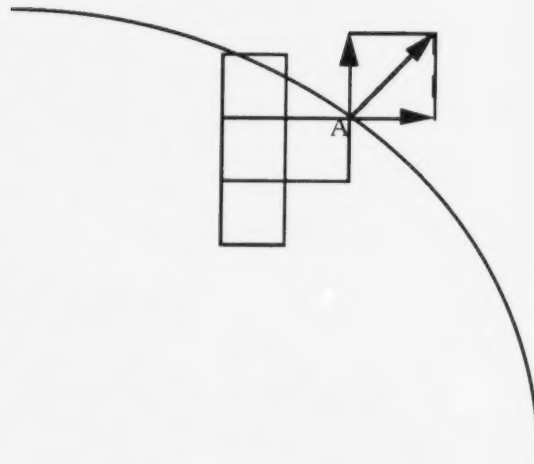


FIG. 4.8. Ray tracing cartoon. A ray follows the gradient (normal) direction of the wavefront at a given point.

Figure 4.9 shows the horizontal layered velocity model which has been used in Chapter 3 (Figure 3.8 (a)). Setting the borehole axis at zero offset, Figure 4.10 (a) illustrates contours of the calculated first arrival traveltimes for waves generated by a surface source located at 250 meters offset. The dashed lines in Figure 4.10 (a) indicate three major interfaces illustrated in the model (Figure 4.9). Note that head waves reach the borehole first between the depths from 120 to 160 meters as a result of the strong refraction by the high velocity carbonate layer. Ray tracings are then carried out by following the steepest descent direction (Figure 4.8) through these computed traveltimes from each receiver back to the source. Figure 4.10 (b) illustrates the ray paths obtained by this technique for borehole receivers at a 10 meter spacing and surface sources located at 250, 150, and 50 meters offset, respectively. Note again the up-going rays for the far offset (250 m) source and deep depth receiver (in a depth from 120 to 160 m) pairs are received at the borehole. The slight differences of the incident critical angle occurring for the rays corresponding to 250 m offset source and 50 m, 60 m, and 70 m depth receiver pairs, respectively, are caused by a defect of our ray tracing codes which need to be improved. However, this ray tracing technique is successful and accurate since the maximum arrival traveltime error between the true arrivals and the arrivals through the traced ray paths is only 0.2%. Moreover, the computational time efficiency and the easily accommodated recording geometry make this technique applicable in curved ray tomographic inversion. For the reflection traveltime contours associated with waves generated by a source located at 250 m offset and then reflected from the carbonate interface (Figure 4.11 (a)), ray paths are illustrated in Figure 4.11 (b) for both borehole and surface receivers at a 10 meter spacing.

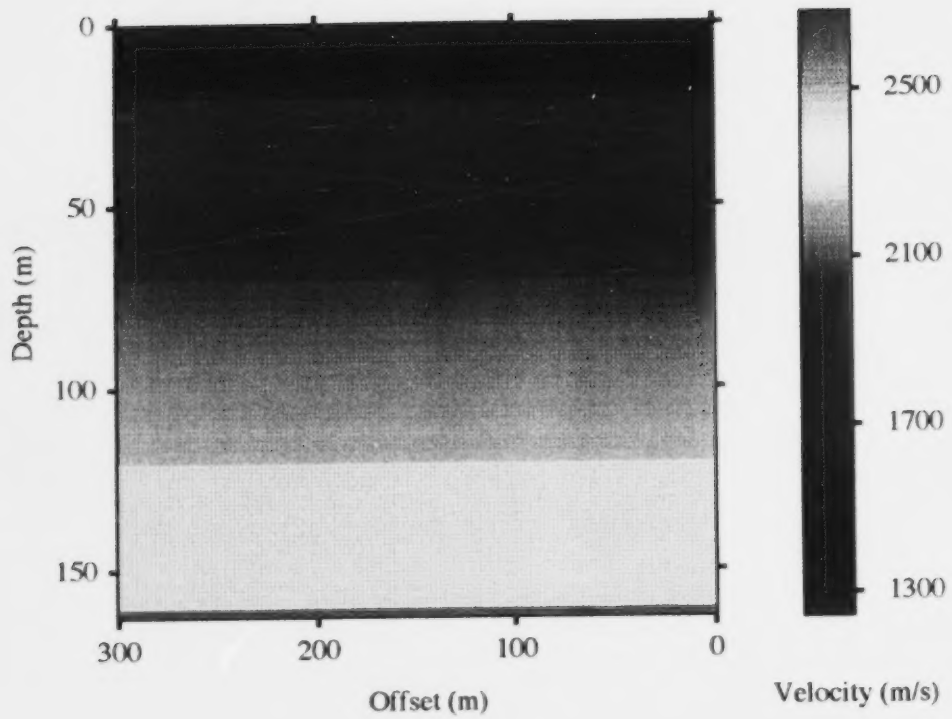


FIG. 4.9. Horizontal layered velocity model. Velocities are coded by varying levels of colours. Different scales are used in horizontal and vertical directions.

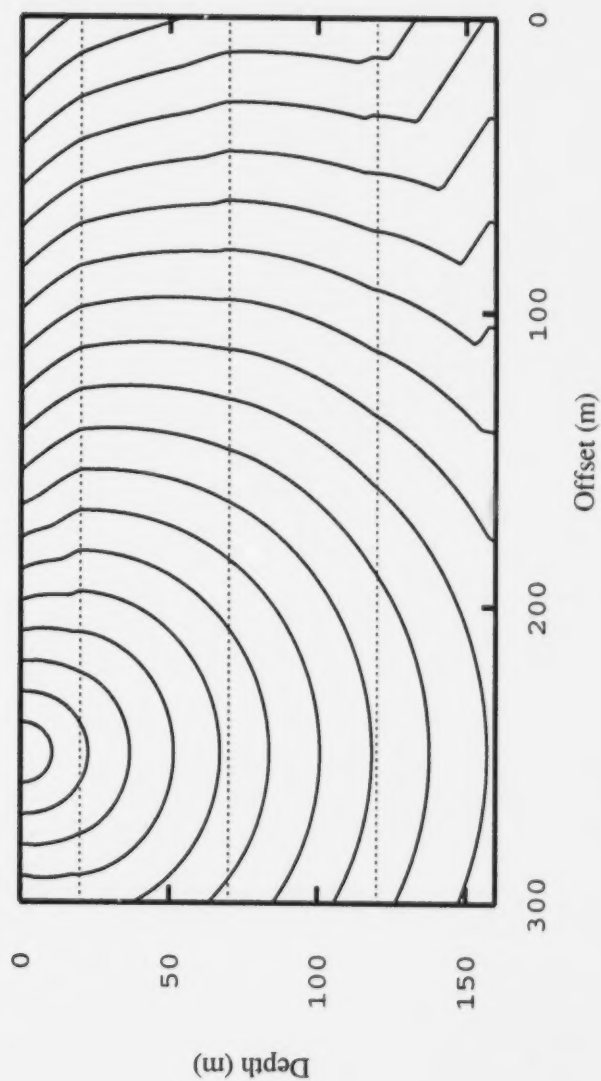


FIG. 4.10 (a). First arrival traveltime contours of the model shown in figure 4.9. The time intervals between the contours are 8.1 ms. The source is located at 250 m offset. The dashed lines represent the major interfaces.

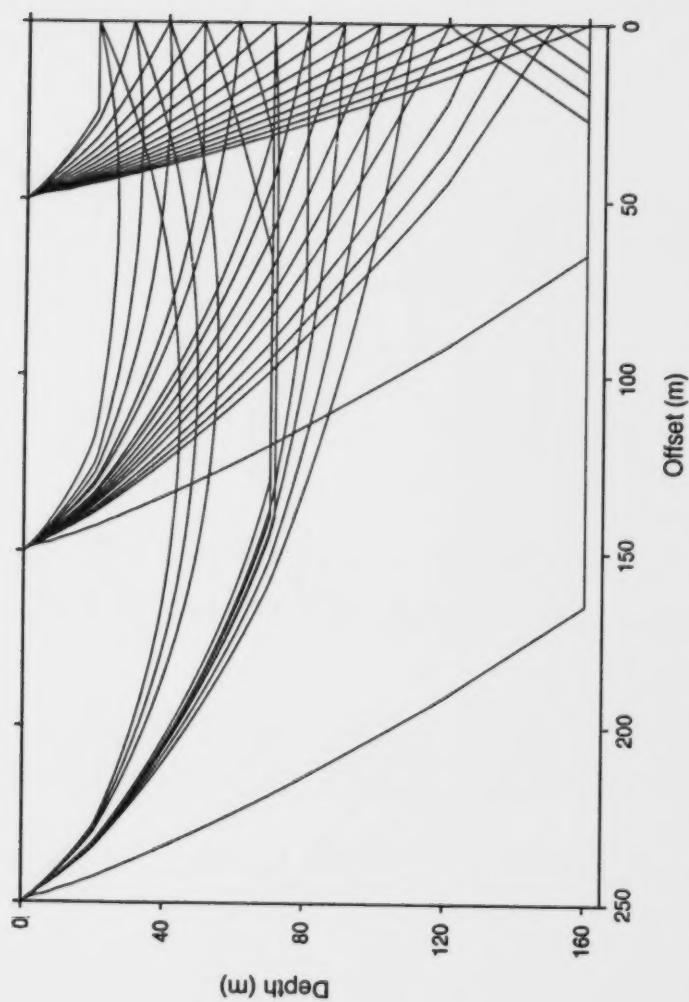


FIG. 4.10 (b). Ray paths of the first arrival energy for the model shown in figure 4.9. Three sources are located at 250 m, 150 m, and 50 m offset, respectively. Receivers are located in the borehole at a 10 m spacing.

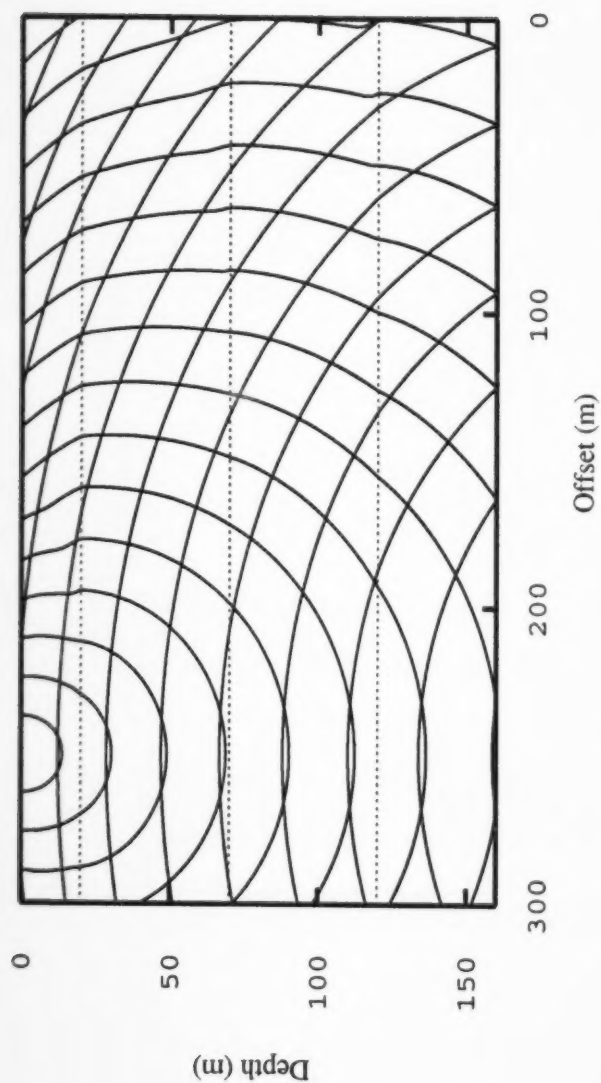


FIG. 4.11 (a). Traveltime contours of the incident and reflected waves for the model shown in figure 4.9. The time intervals between the contours are 10.2 ms. The source is located at 250 m offset. The dashed lines represent the major interfaces.

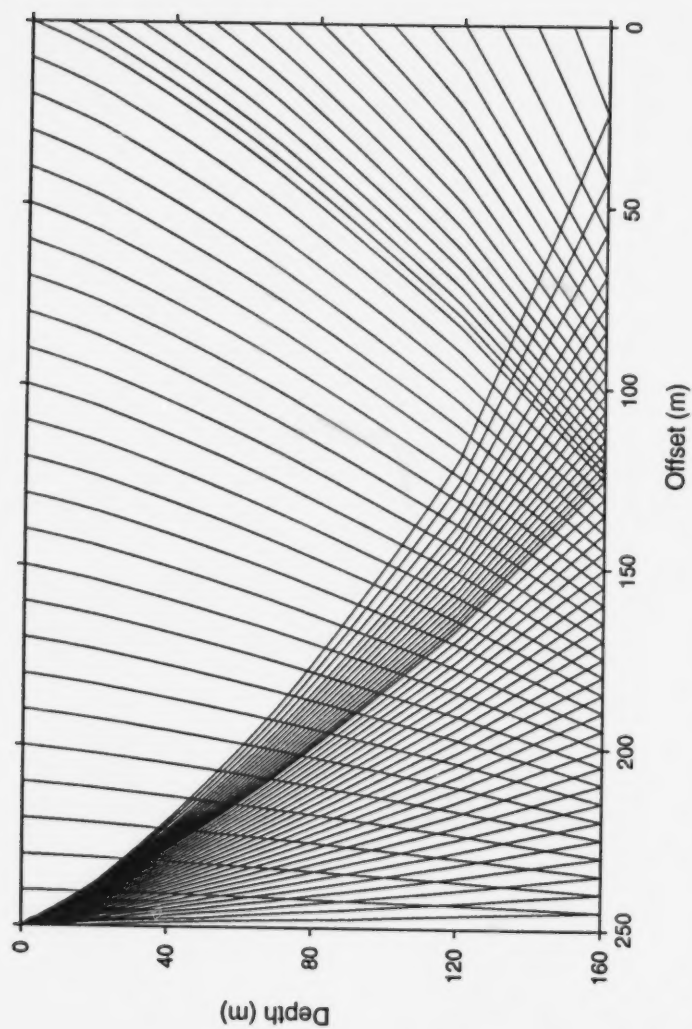


FIG. 4.11 (b). Ray paths of the energy reflected from the high velocity carbonate interface shown in figure 4.9. The source is located at 250 m offset. Both surface and borehole receivers are located at a 10 m spacing.

The model illustrated in Figure 4.12 is constructed by embedding a rectangular low velocity block into the previous model shown in Figure 4.9. This block is located 50 m from the borehole axis and horizontally extended to 200 m offset with 20 m thickness and 10 m away from both the top and the bottom interfaces of the layer (Figure 4.12). The block has a velocity of 2080 m/s which gives a velocity difference of 360 m/s from that of the background. The first arrival traveltimes contours for a source at 250 m offset and the traced ray paths for the sources at 50 m, 150 m, and 250 m offset, respectively, are shown in Figures 4.13 (a) and (b), respectively. Note the occurrence of the planar wavefronts within the block generated by the headwaves propagating along the top interface of the low velocity block (Figure 4.13 (a)) and the downward diving ray associated with the 150 m source and 160 m receiver pair when the ray encounters this low velocity block (Figure 4.13 (b)). For a source at 250 m offset, Figures 4.14 (a) and (b) illustrate the wavefronts for the incident and associated reflected waves from the carbonate interface and the traced ray paths for both surface and borehole receivers at a 10 m spacing. Note that the rays received at surface receivers ranging from 130 m to 160 m offset (Figure 4.14 (b)) are diffractions from the upper left hand corner of the block after they are reflected from the carbonate interface; similarly for the rays received by borehole receivers at depths from 90 m to 120 m. In another case, the received rays at surface 90 m, 100 m, and 110 m offsets are reflections after they diffracted from the lower left hand corner of this block (Figure 4.14 (b)), respectively.

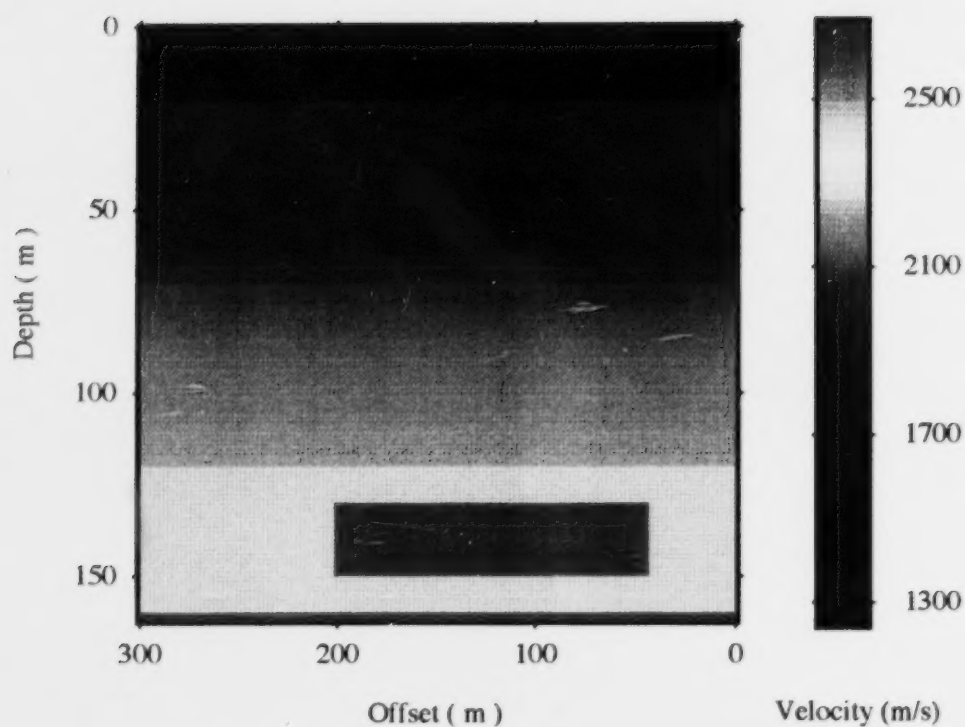


FIG. 4.12. A velocity structure used for reflection VSP tomographic inversion study. This model is constructed by embedding a low velocity block in the previous structure shown in figure 4.9.

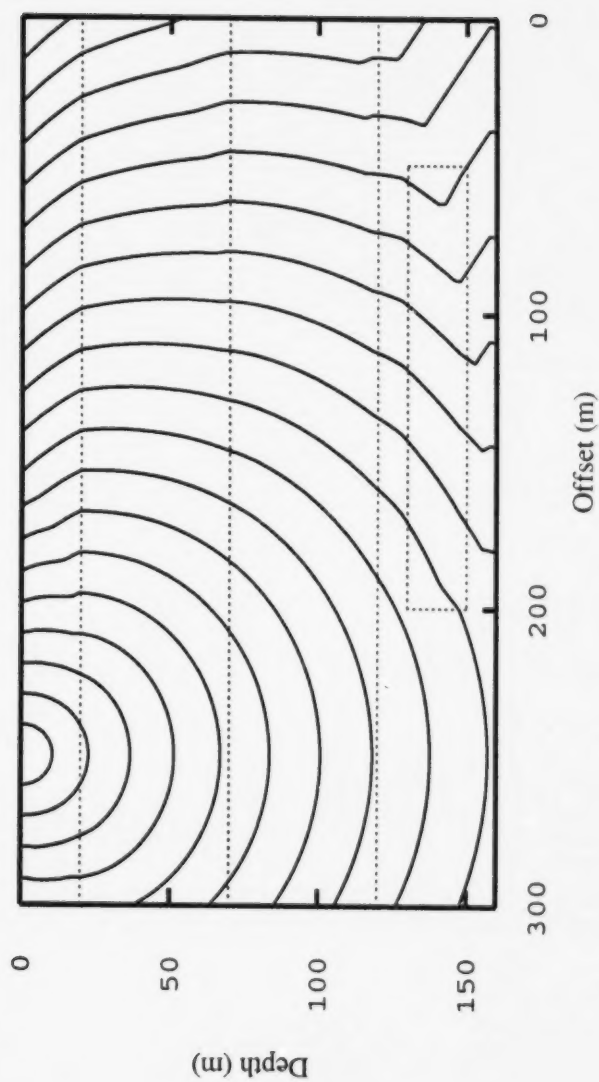


FIG. 4.13 (a). First arrival traveltimes contours of the model shown in figure 4.12. The time intervals between the contours are 8.1 ms. The source is located at 250 m offset. The velocity anomaly is represented by the dashed rectangle.

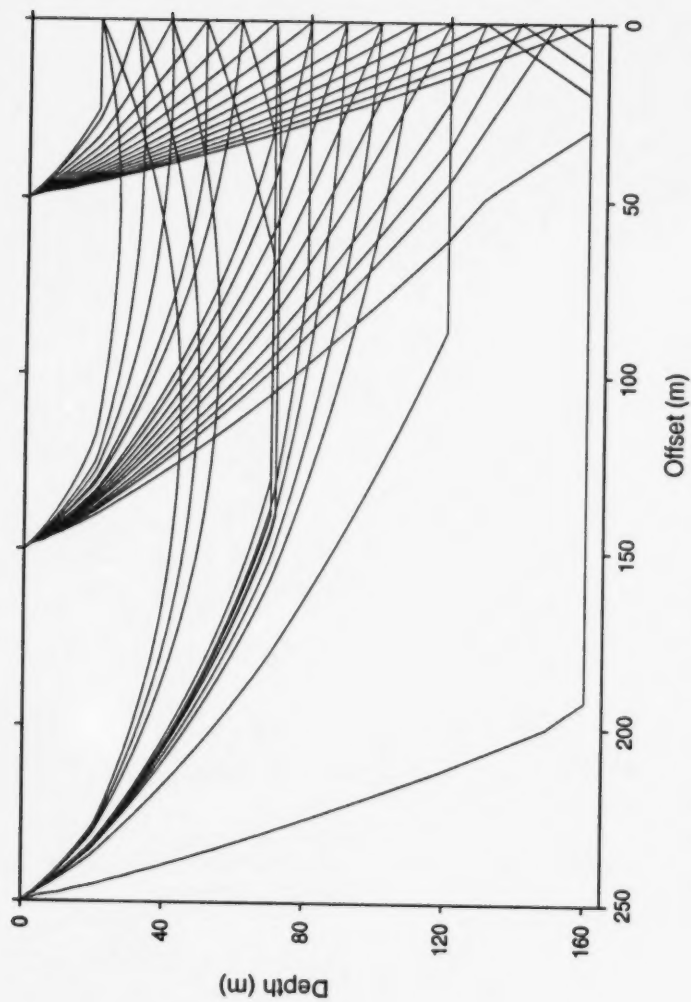


FIG. 4.13 (b). Ray paths of the first arrival energy for the model shown in figure 4.12. Three sources are located at 250 m, 150 m, and 50 m offset, respectively. Receivers are located in the borehole at a 10 m spacing.

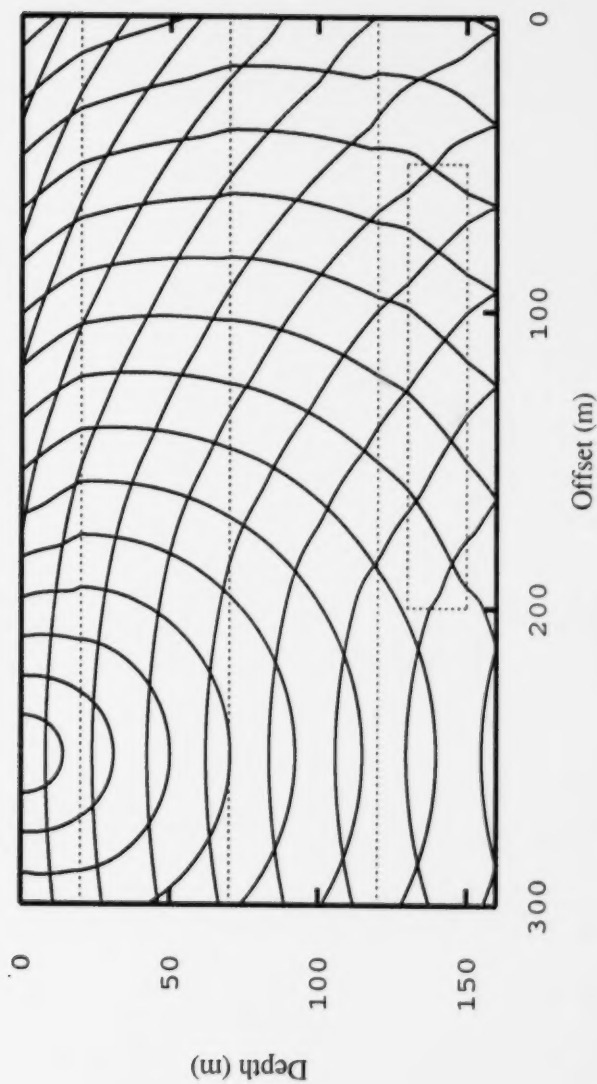


FIG. 4.14 (a). Traveltime contours of the incident and reflected waves for the model shown in figure 4.12. The time intervals between the contours are 10.4 ms. The source is located at 250 m offset. The velocity anomaly is represented by the dashed rectangle.

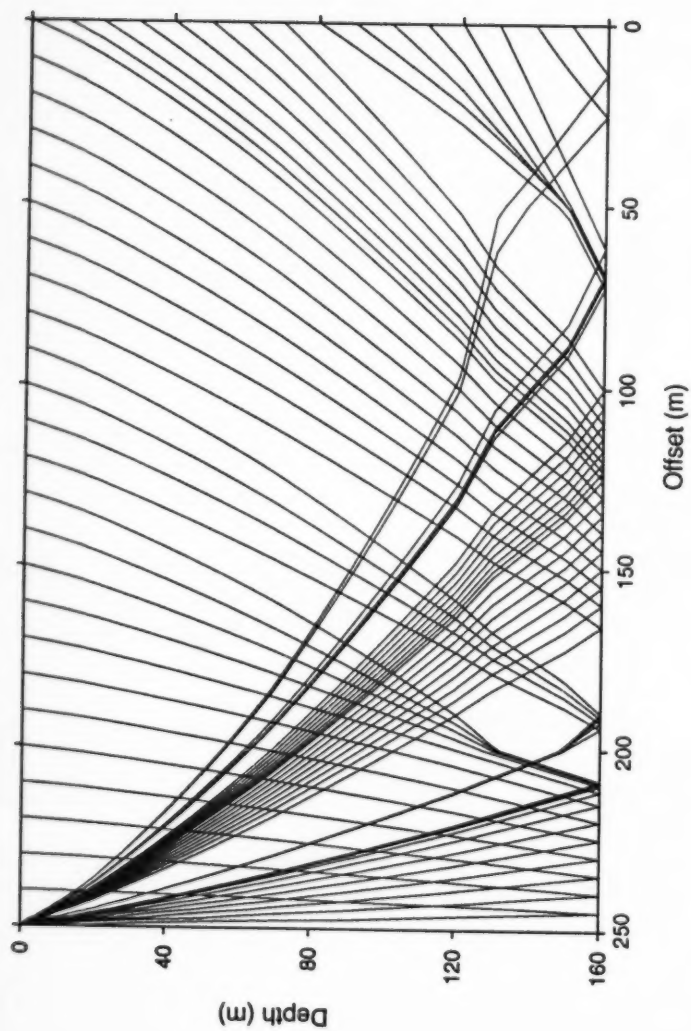


FIG. 4.14 (b). Ray paths of the energy reflected from the high velocity carbonate interface shown in figure 4.12. The source is located at 250 m offset. Both surface and borehole receivers are located at a 10 m spacing.

Overall, this newly developed ray tracing technique provides an efficient method applicable to curved ray traveltime tomographic inversion. This method also yields the ray paths of all wave types that comprise first arrivals as well as reflected arrivals (body waves, headwaves and diffractions) as noted earlier (Figures 4.13 (b) and 4.14 (b)). Moreover, an arbitrary recording geometry is easily accommodated since arrival times are calculated throughout the velocity field.

4.4.2 A Simple Synthetic VSP Tomography Study Using Curved Ray Paths and Reflections

As noted earlier, traveltime tomographic inversion is a nonlinear problem since a ray path is guided by the unknown velocity structure. However, this difficulty is often remedied by assuming a reference velocity structure which guides ray paths. The time delays are used to obtain the velocity perturbations (equation 4.9) and the perturbations are then used to update the reference velocity. This process is iterated till the best match in a least square sense between the reconstructed velocity image and the geology is found.

To test the effect of reflected rays on the improvement of the image quality, a simple synthetic VSP tomographic inversion is carried out for the model illustrated in Figure 4.12. In the synthetic survey, geophones are laid out with a 10 m spacing both along a radial line extending from the borehole axis to an offset of 290 m and down in the borehole to a depth of 160 m; the 29 surface sources are set at the positions identical to the surface geophones except the one at zero offset. In the forward modeling, the traveltimes of the first arrivals and reflected from the carbonate interface are calculated for all

the sources and associated geophones using the methods described in Chapter 3. This gives a total of 435 first and 899 reflected arrivals. The velocity structure illustrated in Figure 4.9 is used as the reference velocity and ray paths are obtained through this reference using the newly developed ray tracing technique. These inputs with 10 m x 10 m image pixels set up an inversion problem with 1334 equations and 464 unknowns. The algebraic reconstruction technique (ART) is used to solve this over determined inversion problem.

The tomographic inversion is first intended to carry out only using first arrival traveltimes. The ray distribution for this scenario is illustrated in Figure 4.15 and indicates that it is impossible to reconstruct the region of interest, i.e., the low velocity block which might mimic a steam recovery zone, since the ray coverage is extremely poor in this region as the result of the strong refraction from the high velocity carbonate structure immediately below it. This effect is also indicated through the ray tracing process in Figure 4.10 (b).

To resolve a reliable geological structure, a wide range of illumination angles is always required (Bregman et al., 1989b). As a result, it is natural to attempt to use the additional information contained in the reflections from the carbonate interface. Figure 4.16 illustrates the new ray distribution containing both rays for the first arrivals and rays for the reflections from the carbonate. Relatively dense rays cover most of the structure, especially the region of interest as indicated by the high color code (Figure 4.16). Since the rays associated with the headwaves do not contain any more information than those contained in reflections, only 381 first arrivals are used in the inversion in order to reduce the computation load. The resulting velocity

image is illustrated in Figure 4.17. Compared to the original model (Figure 4.12), the resulting image presents a reasonable structure, especially as the two vertical edges of the low velocity block are readily seen. The smearing effect surrounding the upper and lower interfaces of the low velocity block and the false high velocity image above it result from a lack of horizontally distributed rays in this region. The model misfit E is 0.130 ms and the relative rms velocity misfit is 5.6 percent between the resulting image and the model. The relative rms velocity misfit Δv is written as

$$\Delta v = \sqrt{\frac{1}{n} \sum_{j=1}^n \left(\frac{v_j^{mod} - v_j^{img}}{v_j^{mod}} \right)^2} \quad (4.23)$$

where v_j^{mod} and v_j^{img} are model and resulting image velocities for j th pixel; n is the total number of image pixels. The computational time required for inversion is 35 minutes of CPU time on the University of Alberta CONVEX computer.

An advantage of VSP tomography relative to surface seismic is that the energy travels shorter distances through the earth and only once through the unconsolidated near-surface and, as a result the signals are less attenuated (Squires, 1992; Docherty, 1992) especially at high frequencies. Crosshole tomography is expected to provide even better resolution than VSP since most of the wave energy does not travel through the highly attenuating near-surface and the travel paths are shorter. In addition, the resolution of crosshole tomography is not depth limited since most of the energy travels between wells. However, field surveys for crosshole tomography are very expensive, which makes VSP tomography practically compatible. Further, crosshole tomography can image only the planar region between two wells whereas VSP tomography can obtain additional 3-D information due to the

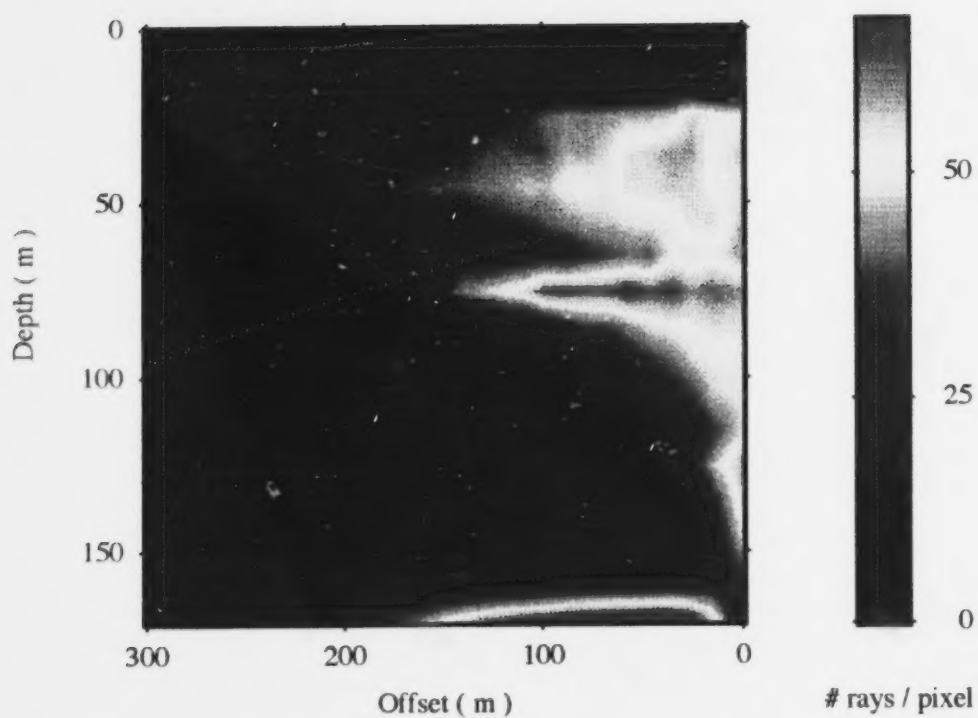


FIG. 4.15. The density map of rays associated with the first arrivals. The raypaths are traced from the reference velocity structure shown in figure 4.9. The numbers of the rays that pass through each pixel of the structure are coded by varying levels of colours.

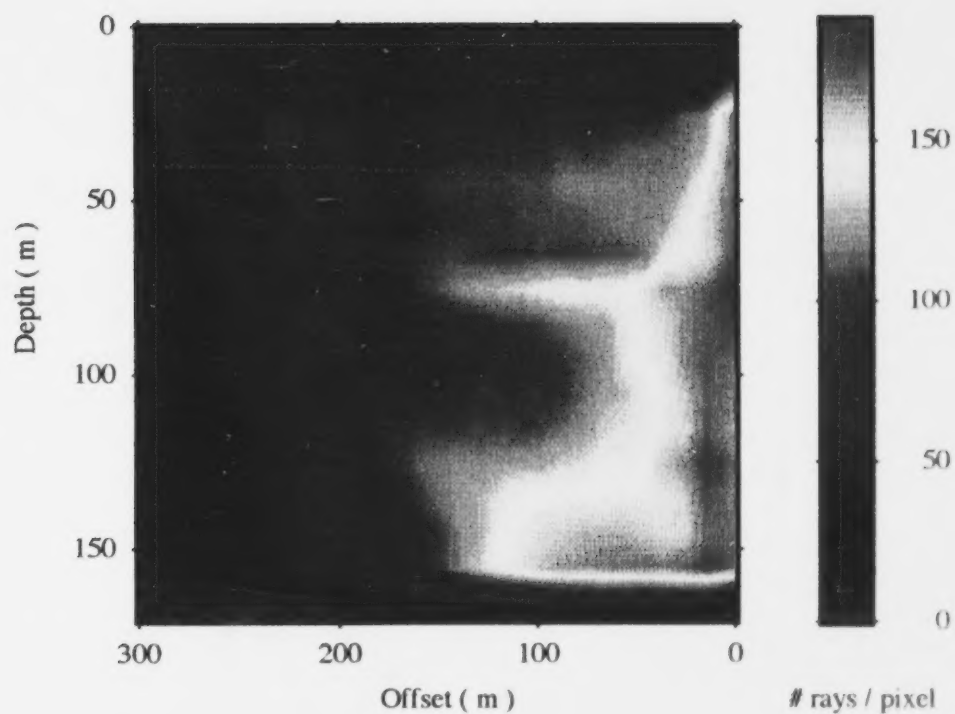


FIG. 4.16. The density map of rays associated with the first and reflected arrivals. The raypaths are traced from the reference velocity structure shown in figure 4.9. The numbers of the rays that pass through each pixel of the structure are coded by varying levels of colours.

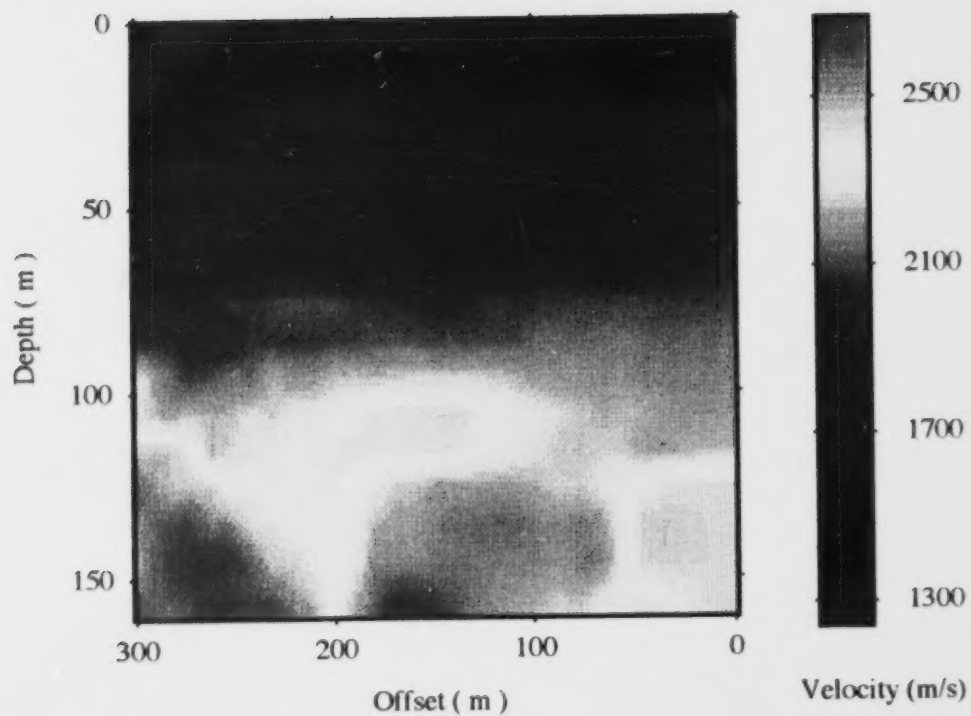


FIG. 4.17. Reconstructed velocity image using ART method. Both the first and reflected arrivals are used to reconstruct the image. The velocity structure shown in figure 4.9 is used as the reference.

fact that one can move azimuthally around the wellbore. Therefore, the VSP tomography may be a useful tool in seismology

In this chapter, three different inversion techniques were discussed and compared by applying them to a hypothetical geological structure. A preliminary study of the effect of the angular ray coverage on a resulting image is also carried out by taking the advantage of the strong reflection from the high velocity carbonate layer. The newly developed traveltimes calculation method (Chapter 3) as well as the associated ray tracing technique are useful tools for other seismic applications such as tomography because of the efficiency and accuracy. However, there two very important issues that were not given sufficient attention in above discussions. First the validity and reliability of traveltimes tomography rely on the applicability of ray theory. Since the ray theory is based on the high frequency assumption, the scale of velocity changes must be much larger than the typical wavelength used in tomographic inversion. For example, Williamson (1991) and his co-workers (Williamson et al., 1993) indicate that the finest structure that ray tomographic inversion can resolve is not smaller than the first Fresnel zone. Consequently, the velocity discontinuities occurring on a realistic geological structure are often smeared in the resulting image (Pratt et al., 1991). Second, the ray tracing here follows the steepest descent direction of the obtained wavefronts or the traveltimes. At present, the fastest traveltimes are calculated both for the first arrival traveltimes.(e.g., Vidale, 1988, 1990; Podvin et al., 1991; Moser et al., 1991; Van Trier et al., 1991; Schneider et al., 1992 and Coultrip, 1993) or the fastest reflected and/or diffracted arrival traveltimes (Chapter 3) and there is no distribution of the actual energy taken into account. In practice, however, it is the energy of a seismic arrival that is

observed and is consequently most interesting. By compressing the seismic section into a series of traveltimes, much important information has been lost. Moreover, the first arrival and the fastest reflected and/or diffracted arrivals often have the maximum geometric spreading effect. For example, the fastest arrivals observed at the surface are all the diffractions from the upper corner of the step fault as illustrated in Figures 3.15 (c) and (d). As a result, the seismic energy carried by these fastest arrivals can be very small and possibly difficult to identify for a large scale survey in a complex geological structure. Geoltrain et al. (1993) provided detailed discussions on how these calculated traveltimes are misleading in some advanced seismic processing techniques (e.g., Kirchhoff migration).

However, as noted earlier, and despite the limitations, traveltime tomographic inversion and the newly developed traveltime calculation techniques are still very useful in many fields.

Chapter 5 Conclusions

This thesis focuses on the analysis of seismic traveltimes which includes traveltime calculation and traveltime tomography. The theoretical basis of the work are primarily the eikonal equation and Fermat's principle which are applicable for seismic waves propagating in weak inhomogeneous media under a high frequency approximation.

There are numerous methods of traveltime calculation. The conventional methods are the shooting or the bending methods which are not computational efficient and have the problem that the obtained first arrival traveltimes are sometimes ambiguous in a complex geological structure. Recently developed methods focus on calculating the first arrival traveltimes to given points over the entire structure. The obtained wavefronts may include all kinds of arrivals (e.g., head waves and diffractions). Without iterations, ray paths are then obtained by following the steepest descent direction of the wavefronts backwards from receivers to the source.

The new methods have obvious advantages over the conventional ray tracing methods in complex media or when a large number of rays are required. Moreover, arbitrary recording geometries are easily accommodated since the arrival times are obtained everywhere throughout the geological structure. Therefore, these traveltime calculation schemes and the associated ray tracing method are particularly useful in traveltime tomographic inversion where the source-receiver combinations can be complex.

However, these methods can only calculate the first arrival traveltimes, which fundamentally restricts their applications since seismic reflection profiling forms the basis for seismology. The reflection traveltime calculation method developed in this thesis is based on Fermat's and Huygens' principles. With the prior knowledge on the position of a reflector, the reflection traveltimes may be calculated efficiently and accurately. The time required for the calculation of the reflected traveltimes is only twice as that used in the calculation of the first arrival traveltimes. The computational errors are primarily the result of the diffraction calculations and are estimated to be at most 1 per cent of the observed arrival traveltimes. The errors resulting from the boundary mismatching may be reduced by improving the grid geometry applied in a model. Moreover, the velocity model used in our method consists of a set of rectangular blocks of constant velocity. This velocity model is useful and conforms automatically to the configurations used in many tomographic modeling schemes.

One difficulty of these newly developed methods is that there is no distribution of the actual energy that has been taken into account in current study. Only the fastest traveltimes (i.e., the first arrivals or the fastest reflected and/or diffracted arrivals) are obtained by these methods. These arrivals often have the maximum geometric spreading effect. Therefore, the seismic energy carried by these fastest arrivals can be very small and possibly difficult to identify for a large scale survey in a complex geological structure.

The ray tracing results for a velocity structure similar to that encountered in the shallow oil sands formations near Ft. McMurray show that the ray paths are severely refracted or even critically refracted. A straight line ray path assumption is no longer valid for this velocity structure.

Further, head waves propagating along the interface of the high velocity carbonate layer result in a lack of the ray coverage on the region of interest. Therefore, it is impossible to image any velocity structure in this region for a VSP study. However, the high velocity carbonate layer can provide good reflection seismic data which may remedy the above problem.

A dense ray covered synthetic reflection VSP tomographic inversion study is carried out using the inversion algorithm commonly referred to as the algebraic reconstruction technique. The primary goal of the inversion study is to image the heterogeneity embedded in a vertically changing velocity structure. The ray paths are traced for both the first arrivals and the arrivals reflected from the carbonate layer. The arrival traveltimes are calculated for the background velocity structure using the new reflection traveltime calculation method. The results show that the vertical edges of the heterogeneity are readily seen after the velocity reconstruction. The smearing effect for the lateral edges results from a lack of horizontally distributed rays in this region. The model misfit E is 0.130 ms and the relative rms velocity misfit is 5.6 percent between the resulting image and the model.

The inversion study also shows that the parameter of the model misfit E may not be an absolute indicator on the resulting imaging quality, especially for a geological velocity structure with poor ray coverage.

In addition to enhancing the accuracy of the calculations, the future direction of this research will be towards the calculation of the geometrical spreading amplitudes of propagating waves and possibly an estimation of the attenuation effect caused by the anelasticity of a geological structure.

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APPENDIX

The first arrival traveltimes calculation algorithm was first developed by Schneider et al (1992). The new reflection traveltimes calculation method and the C-code computer programs were developed as part of this thesis at the University of Alberta

```
/*
 * SCHNEIDER.C
 *
 * This is the main program of Schneider's method.
 * Supporting codes DOWNSTART.C; DOWNWAVE.C; UPWAVE.C;
 * VELOCITY.C; RTBIS.C; REF.H
 * are needed.
 * The code RTBIS.C may be referred in
 * "Numerical Recipes".
 */

#include <stdio.h>
#include <math.h>

#include "ref.h"

typedef struct DCOMPLEX { double r,im;} dcomplex;

char *filenameA = "Data/velocity.dat",
     *filenameB = "Data/source.dat",
     *filenameC = "Data/direct.dat",
     *filenameD = "Data/arrival.dat",

double w,slow,z1,z2,t1,t2,dd,source,
       t[HGRD+1][2*VGRD+1],v[HGRD+1][2*(VGRD+1)],s[NS+1];
dcomplex xy[HGRD+1][2*VGRD+1];

main()
{
FILE  *output,*outputC,*outputD;
int   i,j,k,k0,ks,ke,kr,sn0;

/*
 * The velocity function of a model for which
 * traveltimes are calculated.
 */

velocity();

output=fopen(filenameA,"w");
for(j=1;j<=VGRD;j++)
{
    for(i=1;i<HGRD;i++)
    {
```

```

        fprintf(output, "%g\n", v[i][j]);
    }
}
fclose(output);

output=fopen(filenameB, "w");
for(i=0; i<NS; i++)
{
    /*
     * Give source positions.
     */
    s[i]=(double)(40*i+2)*DX;
    fprintf(output, "%f\n", s[i]);
}
fclose(output);

/*
 * Set up cell grids
 */

for(j=0; j<=VGRD; j++)
{
    for(i=0; i<=HGRD; i++)
    {
        xy[i][j].r=i*DX;
        xy[i][j].im=j*DY;
    }
}

outputC=fopen(filenameC, "w");
outputD=fopen(filenameD, "w");

/*
 * Source loop = source number: NS total.
 */

for(i=0; i<NS; i++)
{
    source=s[i]; sn0=(int)(source/DX);

    /*
     * Give initial values to the time array.
     * t[model-collum][model-row]
     */

    for(j=0; j<=VGRD; j++)
    {
        for(k=0; k<=HGRD; k++)
        {
            t[k][j]=999;
        }
    }

    /*
     * Set up initials for further calculation
     */

    ks=0;

```



```

downstart(ks);

/*
 * Downgoing waves
 */

ks=1;ke=VGRD;
downwave(ks,ke,v,t);

/*
 * Upgoing waves
 */

ks=VGRD;ke=0;
upwave(ks,ke,v,t);

/*
 * Output traveltimes & first arrivals
 */

for(k=0;k<=VGRD;k++)
{
    for(j=2;j<=HGRD;j++)
    {
        fprintf(outputC,"%g %g %g\n",
                (xy[j][k].r-10.),xy[j][k].im,t[j][k]);
    }
    fprintf(outputC,"\n");
}

for(k=0;k<=VGRD-1;k++)
{
    fprintf(outputD,"%g %g\n",xy[HGRD][k].im,t[HGRD][k]);
}
fclose(outputC);
fclose(outputD);
}

```

```

/*
 * DOWNSTART.C
 *
 * This program is used as an initialization
 * of Schneider's method for downgoing
 * propagation waves.
 */

#include <stdio.h>
#include <math.h>
#include "ref.h"

typedef struct DCOMPLEX { double r,im;} dcomplex;

double w,slow,z1,z2,t1,t2,dd,source,
        v[HGRD+1][2*(VGRD+1)],t[HGRD+1][2*VGRD+1];
dcomplex xy[HGRD+1][2*VGRD+1];

double downstart(ks)

int ks;

{
double time,time0,root,rtbis(),func();
int i,j,jp;

    for(j=0;j<HGRD;j++)
    {
        /*
         * Give initial times to the grids adjacent
         * to the source.
         */

        if(source >= xy[j][ks].r && source < xy[j+1][ks].r)
        {
            jp=j;
            z1=source-xy[j][ks].r;
            z2=xy[j+1][ks].r-source;
            t[j][ks]=z1/v[j][ks+1];
            t[j+1][ks]=z2/v[j][ks+1];
            if(z1 < 1.0e-3 )
            {
                if( v[j][ks+1] < v[j-1][ks+1])
                {
                    t[j][ks+1]=DY/v[j-1][ks+1];
                }
                else
                {
                    t[j][ks+1]=DY/v[j][ks+1];
                }
                t[j-1][ks+1]=sqrt(DY*DY+DX*DX)/v[j-1][ks+1];
                t[j+1][ks+1]=sqrt(DY*DY+DX*DX)/v[j][ks+1];
            }
            else
            {
                t[j][ks+1]=sqrt(z1*z1+DY*DY)/v[j][ks+1];
                t[j+1][ks+1]=sqrt(z2*z2+DY*DY)/v[j][ks+1];
            }
        }
    }
}

```

```

    }
    break;
}
}

/*
 * Give initial times to the row/column of the model
 * where the source is located.
 */

for(j=jp+1;j<HGRD;j++)
{
    t[j+1][ks]=t[j][ks]+
        (xy[j+1][ks].r-xy[j][ks].r)/v[j][ks+1];
}
for(j=jp;j>0;j--)
{
    t[j-1][ks]=t[j][ks]+
        (xy[j][ks].r-xy[j-1][ks].r)/v[j-1][ks+1];
}

/*
 * Extend time to the grids on the next row/column
 */

for(j=jp+1;j<HGRD;j++)
{
    z1=xy[j][ks].im;
    z2=xy[j][ks+1].im;
    t1=t[j][ks];
    t2=t[j][ks+1];

    w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
    slow=1/v[j][ks+1];
    dd=DX;

    root=rtbis(func,z1,z2,XACC);

    time0=w*(root*root-z1*z1)+t1*t1;
    time=sqrt(time0)+
        slow*sqrt((z2-root)*(z2-root)+DX*DX);

    if(time < t[j+1][ks+1])
    {
        t[j+1][ks+1]=time;
    }
}

for(j=jp-1;j>0;j--)
{
    z1=xy[j][ks].im;
    z2=xy[j][ks+1].im;
    t1=t[j][ks];
    t2=t[j][ks+1];

    w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
    slow=1/v[j-1][ks+1];

```

```
dd=DX;

root=rtbis(func,z1,z2,XACC);

time0=w*(root*root-z1*z1)+t1*t1;
time=sqrt(time0)+
      slow*sqrt((z2-root)*(z2-root)+DX*DX);

if(time < t[j-1][ks+1])
{
    t[j-1][ks+1]=time;
}
}
```

```

/*
 * DOWNWAVE.C
 *
 * The program used for calculating the traveltimes
 * associated with downgoing waves.
 * The cases in this code correspond to the calculation
 * steps illustrated in figure 3.3 in the text.
 */

#include <stdio.h>
#include <math.h>
#include "ref.h"

typedef struct DCOMPLEX { double r,im;} dcomplex;

double w,slow,z1,z2,t1,t2,dd,source;
dcomplex xy[HGRD+1][2*VGRD+1];

double downwave(ks,ke,V,T)

double V[HGRD+1][2*(VGRD+1)],T[HGRD+1][2*VGRD+1];
int ks,ke;
{
double time,time0,root,rtbis(),func();
int i,j,k;

for(k=ks;k<ke;k++)
{
/*
 * case I: waves propagate to the right.
 */

for(j=0;j<HGRD;j++)
{
z1=fabs(xy[j][k].r-source);
z2=fabs(xy[j+1][k].r-source);
t1=T[j][k];
t2=T[j+1][k];

w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
slow=1/V[j][k+1];
dd=DY;

root=rtbis(func,z1,z2,XACC);

if(root != 0)
{
time0=w*(root*root-z1*z1)+t1*t1;
time=sqrt(time0)+
slow*sqrt((z2-root)*(z2-root)+DY*DY);

if(time < T[j+1][k+1])
{
T[j+1][k+1]=time;
}
}

if(V[j][k+1] >= V[j+1][k+1])

```

```

        {
            time=T[j+1][k]+DY/V[j][k+1];
        }
    else
    {
        time=T[j+1][k]+DY/V[j+1][k+1];
    }
    if(time < T[j+1][k+1])
    {
        T[j+1][k+1]=time;
    }

    time=t1+sqrt(DX*DX+DY*DY)/V[j][k+1];
    if(time < T[j+1][k+1]) {
        T[j+1][k+1]=time;
    }
}

/*
 * Case II: waves propagate to the left.
 */

for(j=HGRD;j>0;j--)
{
    z1=fabs(xy[j][k].r-source);
    z2=fabs(xy[j-1][k].r-source);
    t1=T[j][k];
    t2=T[j-1][k];

    w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
    slow=1/V[j-1][k+1];
    dd=DY;

    root=rtbis(func,z1,z2,XACC);
    if(root != 0)
    {
        time0=w*(root*root-z1*z1)+t1*t1;
        time=sqrt(time0)+
            slow*sqrt((z2-root)*(z2-root)+DY*DY);

        if(time < T[j-1][k+1])
        {
            T[j-1][k+1]=time;
        }
    }

    time=t1+sqrt(DX*DX+DY*DY)/V[j-1][k+1];
    if(time < T[j-1][k+1]) {
        T[j-1][k+1]=time;
    }
}

/*
 * Case III: propagating right-down wards.
 */

for(j=0;j<HGRD;j++)

```

```

{
    z1=xy[j][k].im;
    z2=xy[j][k+1].im;
    t1=T[j][k];
    t2=T[j][k+1];

    w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
    slow=1/V[j][k+1];
    dd=DX;

    root=rtbis(func,z1,z2,XACC);

    if(root != 0)
    {
        time0=w*(root*root-z1*z1)+t1*t1;
        time=sqrt(time0)+
            slow*sqrt((z2-root)*(z2-root)+DX*DX);

        if(time < T[j+1][k+1])
        {
            T[j+1][k+1]=time;
        }
    }

    if(V[j][k+2] > V[j][k+1])
    {
        time=T[j][k+1]+DX/V[j][k+2];
        if(time < T[j+1][k+1])
        {
            T[j+1][k+1]=time;
        }
    }
}

/*
 * Case IV: propagating left-down wards.
 */

for(j=HGRD;j>0;j--)
{
    z1=xy[j][k].im;
    z2=xy[j][k+1].im;
    t1=T[j][k];
    t2=T[j][k+1];

    w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
    slow=1/V[j-1][k+1];
    dd=DX;

    root=rtbis(func,z1,z2,XACC);

    if(root != 0)
    {
        time0=w*(root*root-z1*z1)+t1*t1;
        time=sqrt(time0)+
            slow*sqrt((z2-root)*(z2-root)+DX*DX);
    }
}

```

```
    if(time < T[j-1][k+1])
    {
        T[j-1][k+1]=time;
    }
}

if(V[j-1][k+2] > V[j-1][k+1])
{
    time=T[j][k+1]+DX/V[j-1][k+2];
    if(time < T[j-1][k+1])
    {
        T[j-1][k+1]=time;
    }
}
}
}
```



```

/*
 * UPWAVE.C
 *
 * The program used for calculating the traveltimes
 * associated with upgoing waves.
 * The cases in this code correspond to the calculation
 * steps illustrated in figure 3.3 in the text.
 */

#include <stdio.h>
#include <math.h>
#include "ref.h"

typedef struct DCOMPLEX { double r,im;} dcomplex;

double w,slow,z1,z2,t1,t2,dd,source;
dcomplex xy[HGRD+1][2*VGRD+1];

double upwave(ks,ke,V,T)

double V[HGRD+1][2*(VGRD+1)],T[HGRD+1][2*VGRD+1];
int ks,ke;
{
double time,time0,root,rtbis(),func();
int i,j,k;

for(k=ks;k>ke;k--)
{
    /*
     * Case VIII: waves propagate to the right.
     */

    for(j=0;j<HGRD;j++)
    {
        z1=fabs(xy[j][k].r-source);
        z2=fabs(xy[j+1][k].r-source);
        t1=T[j][k];
        t2=T[j+1][k];

        w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
        slow=1./V[j][k];
        dd=DY;

        root=rtbis(func,z1,z2,XACC);

        if(root != 0)
        {
            time0=w*(root*root-z1*z1)+t1*t1;
            time=sqrt(time0)+
                slow*sqrt((z2-root)*(z2-root)+DY*DY);

            if(time < T[j+1][k-1])
            {
                T[j+1][k-1]=time;
            }
        }

        if(V[j][k] >= V[j+1][k])

```

```

        {
            time=T[j+1][k]+DY/V[j][k];
        }
    else
    {
        time=T[j+1][k]+DY/V[j+1][k];
    }
    if(time < T[j+1][k-1])
    {
        T[j+1][k-1]=time;
    }

    time=t1+sqrt(DX*DX+DY*DY)/V[j][k];
    if(time < T[j+1][k-1]) {
        T[j+1][k-1]=time;
    }
}

/*
 * Case IX : waves propagate to the left.
 */

for(j=HGRD;j>0;j--)
{
    z1=fabs(xy[j][k].r-source);
    z2=fabs(xy[j-1][k].r-source);
    t1=T[j][k];
    t2=T[j-1][k];

    w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
    slow=1./V[j-1][k];
    dd=DY;

    root=rtbis(func,z1,z2,XACC);

    if(root != 0)
    {
        time0=w*(root*root-z1*z1)+t1*t1;
        time=sqrt(time0)+
            slow*sqrt((z2-root)*(z2-root)+DY*DY);

        if(time < T[j-1][k-1])
        {
            T[j-1][k-1]=time;
        }
    }

    time=t1+sqrt(DX*DX+DY*DY)/V[j-1][k];
    if(time < T[j-1][k-1]) {
        T[j-1][k-1]=time;
    }
}

/*
 * Case X : propagating right-up-wards.
 */

for(j=0;j<HGRD;j++)

```

```

{
    z1=xy[j][k].im;
    z2=xy[j][k-1].im;
    t1=T[j][k];
    t2=T[j][k-1];

    w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
    slow=1./V[j][k];
    dd=DX;

    root=rtbis(func,z1,z2,XACC);

    if(root != 0)
    {
        time0=w*(root*root-z1*z1)+t1*t1;
        time=sqrt(time0)+
            slow*sqrt((z2-root)*(z2-root)+DX*DX);

        if(time < T[j+1][k-1])
        {
            T[j+1][k-1]=time;
        }
    }

    if(V[j][k-1] > V[j][k])
    {
        time=T[j][k-1]+DX/V[j][k-1];
        if(time < T[j+1][k-1])
        {
            T[j+1][k-1]=time;
        }
    }
}

/*
 * Case XI : propagating left-up-wards.
 */

for(j=HGRD;j>0;j--)
{
    z1=xy[j][k].im;
    z2=xy[j][k-1].im;
    t1=T[j][k];
    t2=T[j][k-1];

    w=(t2*t2-t1*t1)/(z2*z2-z1*z1);
    slow=1./V[j-1][k];
    dd=DX;

    root=rtbis(func,z1,z2,XACC);

    if(root != 0)
    {
        time0=w*(root*root-z1*z1)+t1*t1;
        time=sqrt(time0)+
            slow*sqrt((z2-root)*(z2-root)+DX*DX);

        if(time < T[j-1][k-1])

```

```
        {
            T[j-1][k-1]=time;
        }
    }

    if(V[j-1][k-1] > V[j-1][k])
    {
        time=T[j][k-1]+DX/V[j-1][k-1];
        if(time < T[j-1][k-1])
        {
            T[j-1][k-1]=time;
        }
    }
}

}
```

```
/*
 * REF.H
 *
 * A parameter file.
 */

#define HGRD /* number of the horizontal grids */
#define VGRD /* number of the vertical grids */
#define DX /* dimension of the horizontal grid */
#define DY /* dimension of the vertical grid */
#define XACC /* required calculation accuracy */
#define RFT /* vertical position of the reflector
 * in grid unit
 */

#define DIP /* horizontal position of the
 * reflector in grid unit
 */

#define NS /* number of sources in all
 * the calculations
 */

#define V1 /* velocity reference at certain depth */
#define V2
#define V3
#define V4
#define V5
#define V6
#define V7
```

```

/*
 * REFLECTION.C
 *
 * This is the main program of the new reflection
 *   traveltime
 * calculation method and written in January, 1993.
 * Supporting codes DOWNSTART.C; DOWNWAVE.C; UPWAVE.C;
 *                   DOWNWAVE0.C; UPWAVE0.C; RTBIS.C;
 *                   VELOCITY.C; REF.H are needed.
 * The code RTBIS.C may be referred in
 *   "Numerical Recipes".
 */

#include <stdio.h>
#include <math.h>

#include "ref.h"

typedef struct DCOMPLEX { double r,im;} dcomplex;

char *filenameC = "Data/direct.dat",
     *filenameD = "Data/arrivald.dat",
     *filenameG = "Data/reflect.dat",
     *filenameH = "Data/arrivalr.dat",
     *filenameP = "Data/incident.dat";

double w,slow,z1,z2,t1,t2,dd,source,DY,
        t[HGRD+1][VGRD+1],tr[HGRD+1][VGRD+1],
        v[HGRD+5][VGRD+5],s[NS+1];
dcomplex xy[HGRD+1][VGRD+1];
int kr,kd;

main()
{
FILE *output,*outputC,*outputD,*outputG,*outputH,*outputP;
int i,j,k,ks,ke,sn0;

DY=DY1;
kr=(int)(RFT/DY); kd=(int)(DIP/DX);

for(i=0;i<NS;i++)
{
s[i]=(double)(40*i+2)*DX;
}

velocity();

for(j=0;j<=VGRD;j++)
{
for(i=0;i<=HGRD;i++)
{
xy[i][j].r=(double)i*DX;
xy[i][j].im=(double)j*DY;
}
}

outputC=fopen(filenameC,"w");
outputD=fopen(filenameD,"w");

```

```

outputG=fopen(filenameG,"w");
outputH=fopen(filenameH,"w");
outputP=fopen(filenameP,"w");

/* Source number: NS totally. */

for(i=0;i<NS;i++)
{
    source=s[i]; sn0=(int)(source/DX);
    for(j=0;j<=VGRD;j++)
    {
        for(k=0;k<=HGRD;k++)
        {
            t[k][j]=0.300;
        }
    }

/* Set up initials for further calculation */

    ks=0;
    downstart(ks);

/*    downgoing waves    */

    ks=1;ke=VGRD;
    downwave(ks,ke,t);

    for(j=0;j<=kr-1;j++)
    {
        for(k=0;k<=HGRD;k++)
        {
            tr[k][j]=t[k][j];
        }
    }

/*    upgoing waves    */

    ks=VGRD;ke=0;
    upwave(ks,ke,t);

/*    reflected wave initialization    */

    for(j=kr;j<=VGRD;j++)
    {
        for(k=0;k<=HGRD;k++)
        {
            tr[k][j]=0.300;
        }
    }

/*    joined downgoing waves    */

    ks=kr-1;ke=VGRD1;
    downwave0(ks,ke,tr);

/*    pre-reflection    */

    ks=VGRD1;ke=kr;

```

```

upwave0(ks,ke,tr);

ks=kr;ke=0;
upwave(ks,ke,tr);

/* First output data */
for(k=0;k<=VGRD;k++)
{
    for(j=0;j<=HGRD;j++)
    {
        fprintf(outputP,"%g %g %g\n",
                xy[j][k].r,xy[j][k].im,tr[j][k]);
    }
    fprintf(outputP,"\n");
}

/* Re-initialized tr-data */
for(j=0;j<kr;j++)
{
    for(k=0;k<=HGRD;k++)
    {
        tr[k][j]=0.300;
    }
}

j=kr;
for(k=kd+1;k<=HGRD;k++)
{
    tr[k][j]=0.300;
}
for(j=kr+1;j<VGRD1;j++)
{
    for(k=kd+1;k<=HGRD;k++)
    {
        tr[k][j]=0.300;
    }
}

/* reflection */
ks=VGRD1;ke=kr;
upwave0(ks,ke,tr);

ks=kr;ke=0;
upwave(ks,ke,tr);

ks=0;ke=kr-1;
downwave(ks,ke,tr);

ks=kr-1;ke=VGRD1;
downwave0(ks,ke,tr);

for(k=0;k<=VGRD;k++)
{
    for(j=0;j<=HGRD;j++)
    {

```



```

        fprintf(outputC, "%g %g %g\n",
                xy[j][k].r, xy[j][k].im, t[j][k]);
    }
    fprintf(outputC, "\n");
}

for(j=0; j<=HGRD; j++)
{
    fprintf(outputD, "%g %g\n", xy[j][0].r, t[j][0]);
}

for(k=0; k<=VGRD; k++)
{
    for(j=0; j<=HGRD; j++)
    {
        fprintf(outputG, "%g %g %g\n",
                xy[j][k].r, xy[j][k].im, tr[j][k]);
    }
    fprintf(outputG, "\n");
}

for(j=0; j<=HGRD; j++)
{
    fprintf(outputH, "%g %g\n", xy[j][0].r, tr[j][0]);
}
}

fclose(outputC);
fclose(outputD);
fclose(outputG);
fclose(outputH);
fclose(outputP);
}

```